Ministry
of
Education
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Intermediate Division

Mathematics

1977

Draft copy

The draft curriculum guideline for the mathematics program for Grades 7 to 10 is provided as a discussion document. Responses to it will be considered in the summer of 1978, when it will be revised. The final Intermediate Division mathematics guideline will then be produced.

Local school personnel may submit comments and recommendations to the regional offices of the Ministry of Education up to June 1, 1978. Responses may also be forwarded directly to the Intermediate Mathematics Coordinator, Curriculum Branch, Ministry of Education, Mowat Block, Queen's Park, Toronto, Ontario, M7A 1L2.

The Intermediate Division mathematics guideline will be printed in the fall of 1978 for full implementation in Grade 7 in 1979; in Grade 8 in 1980; in Grade 9 in 1981; and in Grade 10 in 1982.

ACKNOWLEDGEMENTS

The Ministry of Education wishes to acknowledge the contributions of the following persons who participated as members of the Intermediate Division Mathematics Guideline Committee and, in particular, those persons who also served as members of the Writing Team.

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The Writing Team was composed of the following members of the above committee:

Jim MacLean; Mike McKenna (part-time); Paul Pogue; George Scroggie; Michael Silbert (part-time); Lorna Strobel (part-time); Peter Weygang; Claire Zeller.

Validation

During the month of February, numerous groups of teachers representing most school boards in the province and other individuals reacted to a preliminary draft of this guideline (excluding Levels 1, 2, and 3). The present draft reflects their responses to the validation exercise. The Ministry of Education wishes to thank the hundreds of teachers and other interested persons who provided input.

PREFACE

This guideline outlines a proposed mathematics program for the Intermediate Division in Ontario. It provides the policy basis for the local development of specific courses for Grades 7 through 10 to meet the needs of different groups within the school population. It contains a rationale for, and description of, the program for the Intermediate Division, as well as specific *Outlines of Topics* for Grades 7 and 8 and for Grades 9 and 10 at five levels of difficulty.

The guideline is supported by resource documents in the form of *Notes For Teachers* that outline possible treatments of the core and optional topics, and by other *Resource Modules* that deal with topics new to the program and include specific examples of materials ready for classroom use by students. At the time of printing of this document, the Notes For Teachers for Levels 1, 2, and 3 and three Resource Modules for Level 3 are complete and are being distributed along with the guideline. Other Resource Modules will follow during the school year 1977-1978, as they are completed.

This guideline and resource materials are to be treated as *draft copies*, for *optional use* during the school years 1977-78 and 1978-79 and for *validation* by teachers and other educators during the school year 1977-78. The Committee and Writing Team will reconvene in 1978 to consider new responses and suggestions from teachers and others and to prepare the official guideline for introduction in the schools of Ontario in September 1979.

Reactions are requested to the present draft of the guideline and to the resource materials. It is expected that teachers throughout the province will study the draft materials carefully and introduce many aspects of the new curriculum into their programs during the coming year. In this way, reactions to the proposed program can be based on personal experiences and the need for further resource materials may be identified.

School boards which have developed their own resource materials related to this program are encouraged to share them with others. By such cooperative action, new aspects of the program can be implemented more effectively.

When revised and distributed to the province, Intermediate Division Mathematics will replace the following documents:

Mathematics I.12A	Grade 7	1963 I41-020
Mathematics I.12B	Grade 8	1964 I41-030
Mathematics I.12C (4 & 5)	Grade 9	1965 I41-040
Mathematics I.12D (4)	Grade 10	1966 I41-050
Mathematics I.12D (5)	Grade 10	1966 I41-051
Curriculum R.P. 27 (Mathematics, Pages 250-254)	Grades 9, 10	1963 I40-010
Curriculum R.P. 31 (Mathematics, Pages 70 & 71)	Grade 10	1963 I82-014
Curriculum R.P. 35 (Mathematics, Years 1 & 2)	Grades 9, 10	1962 I41-090

CONTENTS

RATIONALE

INTRODUCTION

Many studies have been made of the concepts and skills that are fundamental to the development of a coherent mathematics curriculum in the elementary and secondary school continuum. All of these emphasize the unity of mathematics and the desirability of integrating the study of arithmetic, algebra, and geometry throughout the curriculum.

In 1975 the Ministry of Education published *The Formative Years* as its policy statement for planning the curriculum in the Primary and Junior Divisions. In it, mathematics and language are identified as the major elements in the development of communication skills. The mathematics program is described as an integration of concepts and skills within the broad strands of number, measurement, and geometry. This organization is now being extended into the Intermediate Division by this guideline.

It is generally agreed that children understand more than they can verbalize. It follows that a major role of the teacher, through patient and sympathetic listening and careful attention to written work, is to search out the child's meaning and to help him or her to convey it with greater clarity and precision. This applies equally well to students in the Intermediate Division, where greater emphasis should be given to introducing conventional terms when needed and to making sure the student is aware of the need for systematic presentation of solutions. As they use concepts in situations that are familiar to them, the concepts will become better understood and so become the tools with which they can probe, analyse, and communicate qualitative and quantitative ideas of the environment. When the program is better related to the real needs of the pupils and the tasks set take into account both the readiness of the individual and the likelihood of their successful completion, then self-confidence will be enhanced and students will approach mathematics with greater enthusiasm.

An important feature of this new program is the involvement of students in selecting or creating models to represent aspects of the real world; in studying, analysing, and manipulating these models; and finally in relating their findings back to the real world. While attempting to make sense of their world, students will make use of models such as the diagrams of geometry, the numerals and operations of arithmetic, and the graphs and charts that represent data and illustrate relationships. In this way they should develop more confidence in their ability to use the concepts and skills of mathematics, and will be more inclined to appreciate its beauty and power.

AIMS

Speed drills, on both mental arithmetic and paper and pencil calculations, and problems arising from a relatively static, non-technological society were appropriate features of former programs. These drills and problems reflected a time when society was evolving slowly and the pace of technological development was modest. The revised curriculum focuses on mathematics that is basic to life today – a period of considerable complexity and large scale operations – and that anticipates developments in the future. As a consequence, this new curriculum for the Intermediate Division is concerned with applying mathematics to today's world, with a view for the future – rather than the past. It is not concerned with abstract structures of mathematics. This more functional approach implies that modern, powerful mathematical tools should be applied with understanding and discrimination in realistic situations.

- A major aim of the Intermediate Division Mathematics Curriculum is to develop in students the mathematical literacy they will need for productive lives now, and in the future.

The Intermediate Division corresponds with the period of adolescence for the students. The physical and intellectual changes that occur during this period incline the adolescent to think of himself or herself less as a child and more as an adult. At the beginning of the Intermediate Division most students think symbolically when solving practical problems. By the end of the division, many of them will tend toward formal thinking and will have acquired an expanded ability to deal with abstract thought.

As students mature and feel the need for greater personal autonomy, teachers should provide them with more opportunities for self-direction and self-discipline. Given the nature of adolescents, it is vital to provide them with a learning environment in which they can develop their thinking patterns and decision-making skills so that they may grow into responsible, independent adults.

Problem solving is a major goal of mathematical thinking. The ability to pose and solve problems in realistic or pseudo-realistic contexts is a life skill vital to everyone. Students should be encouraged to gather, interpret, and classify relevant data; then to formulate hypotheses and develop skills of inquiry, analysis, synthesis, and evaluation. Emphasis should be placed on both written and oral presentation of solutions. In this way students can reach a full, albeit informal, understanding of logical reasoning.

- A second major aim is to foster in each student the ability to pose and solve problems at a level in keeping with his or her stage of development.

Through a knowledge of the lives and contributions of the great mathematicians and through the diverse nature of its applications, mathematics reveals itself as an integral part of our culture that is fundamental to our technological progress.

Some elementary problems of historical significance in arithmetic, algebra, and geometry could be presented along with the approaches taken to them in earlier civilizations. Materials appropriate to this age level will need to be developed for this purpose.

- A third aim is to develop in students an appreciation of the place of mathematics and its widespread application in our culture.

THE MATHEMATICS PROGRAM

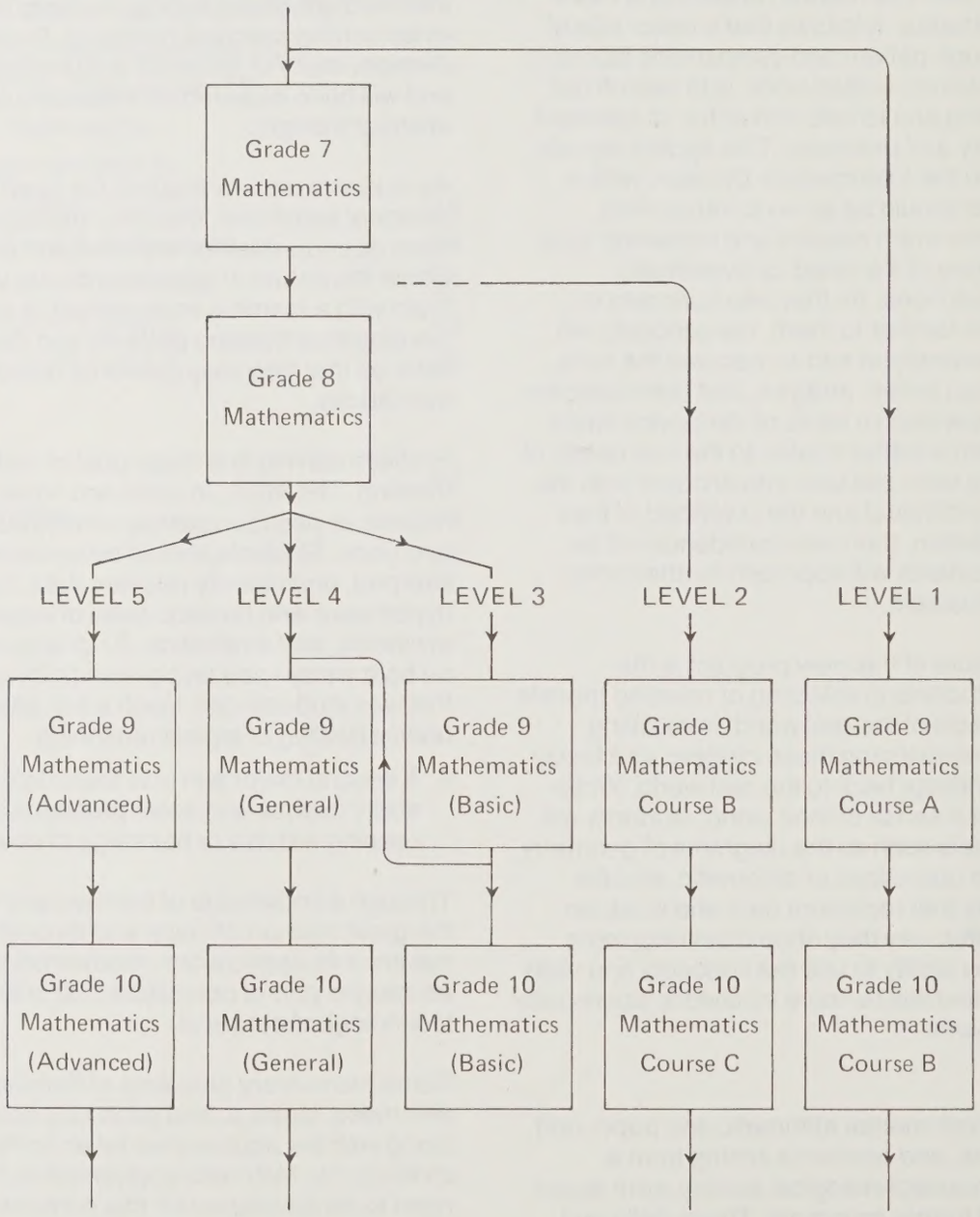
INTRODUCTION

The program of the Primary and Junior Divisions is developed from an integrated view of mathematics in which concepts of number, measurement, and geometry are interwoven with practical experiences and applications of mathematics drawn from the child's world. This experiential approach of the K-6 program is now being extended into the Intermediate Division, with an increased emphasis on consolidating and expanding the basic ideas that were introduced in the earlier years. Throughout the Intermediate Division, students should investigate relevant situations from their world in a search for patterns and for generalizations of their experiences. Their findings should be recorded, discussed, and tested for consistency. At the same time, they should be encouraged to create models to represent their findings (diagrams, numerical expressions, charts, graphs, etc.), then to discuss these models and work with them to develop an expanded understanding of real world situations. The operational skills, which are essential to the growth of mathematical knowledge and to the application of this knowledge to new situations, should be interwoven with and justified by concrete activities and real life applications.

OVERVIEW OF THE PROGRAM

This guideline and the related resource materials are designed to give practical assistance to teachers when building courses that are consistent with the rationale and policy of the guideline. Of major significance are the *Outlines of Topics* that appear later in the document. An overview of the mathematics program, as defined by the Outlines of Topics, is illustrated by the following flow chart and by the brief comments that follow it.

OVERVIEW OF THE MATHEMATICS PROGRAM



The chart illustrates the sequences of courses in the Intermediate Division for which the *Outlines of Topics* have been planned. The topics develop sequentially through Grades 7 and 8 and then within each of the five levels in Grades 9 and 10.

Generally, students should select sequential courses within the same level when making subject choices in the secondary school. However, guided by the counselling and advice of the principal and staff, the final choice of courses is the responsibility of the student and his or her parents. In special circumstances it may be in the best interests of an individual student to take his or her next course at a different level of difficulty.

OVERVIEW OF THE OUTLINES OF TOPICS

The charts on the next five pages provide an overview of the Outlines of Topics for:

- Grades 7 and 8
- Grades 9 and 10, Level 5 (Advanced)
- Grades 9 and 10, Level 4 (General)
- Grades 9 and 10, Level 3 (Basic)
- Grades 9 and 10, Levels 1 and 2

The charts indicate the organization of courses into *Strands* and *Sections* within the strands. Some sections are coded as optional, others as containing some optional topics. The remaining sections and topics are core.

Some sections are coded for delayed implementation in the form in which they are printed. The topics in these sections are new to the Intermediate Division and build sequentially from earlier courses within the Division. It is expected in the interim period that some topics related to these sections will be taught. This is discussed in more detail later in the guideline.

The charts illustrate the sequential development of the major topics from grade to grade within each level and make it easy to compare the topics in the different levels.

GRADE 7 MATHEMATICS

SECTION	NUMERICAL METHODS	ALGEBRA	GEOMETRY
1.	Data Graphs	Communications	*Basic Notions in Geometry
2.	Number Applications	[Equations]	*Accurate Constructions
3.	*Percent	[Relations]	[Patterns with tiles]
4.	*Fractions, Ratios		*Transformations
5.	*Factors		*Enlargements and Reductions
6.	[Integers]		*3-D Geometry
7.	[Other Arithmetics]		
8.	*Measurement of length, area, volume		

GRADE 8 MATHEMATICS

NUMERICAL METHODS	ALGEBRA	GEOMETRY
*Number Applications	*Formulae and Equations	Accurate Constructions
*Fractions, Ratio, Percent	[Relations]	*Congruence †Transformations
*Data Graphs †	[Probability]	*Properties of Plane Figures
Integers	[Flow Charts]	*Dilatation †
[Rational Numbers in Decimal Form]		*3-D Geometry
*Measurement of length, area, volume		
*The circle and related measures		

NOTE:

i) Optional Sections [ ] *indicates sections in which all the topics are optional.*

ii) Optional Topics * *indicates sections in which some of the topics are optional.*

iii) Delayed Implementation † *indicates sections that are to be fully implemented as printed beginning in September 1980.*

LEVEL 5 (ADVANCED)

GRADE 9 MATHEMATICS

SECTION	NUMERICAL METHODS	ALGEBRA	GEOMETRY
1.	Number Applications	*Flow Charts ††	*Circle Applications
2.	*Statistics ††	Equations	*Coordinate ††Geometry
3.	*Powers, Roots	*Polynomials	*Congruence ††Transformations
4.	*Rational Numbers	*Relations	Dilatation
5.	*Applications of ††Percent, Ratio, Measurement	Probability	Displacement
6.	Matrices		

GRADE 10 MATHEMATICS

NUMERICAL METHODS	ALGEBRA	GEOMETRY
Radicals	*Equations and inequations	*Coordinate Geometry
Applied Trigonometry	Relations	*Coordinates and †††Transformations
*Statistics †††	Probability	*Isometries †††
Matrices	Computers	*Deductive †††Geometry
	*Polynomials	Vectors
		*3-D Geometry

NOTE:

i) *Optional Sections* — [ ] *indicates sections in which all the topics are optional.*

ii) *Optional Topics* — * *indicates sections in which some of the topics are optional.*

iii) *Delayed Implementation* — †† *indicates sections that are to be fully implemented as printed beginning in September 1981.*

††† *indicates sections that are to be fully implemented as printed beginning in September 1982.*

LEVEL 4 (GENERAL)

GRADE 9 MATHEMATICS (GENERAL)

SECTION	NUMERICAL METHODS	ALGEBRA	GEOMETRY
1.	Number Applications	*Flow Charts ††	*The Circle
2.	*Statistics ††	*Equations	*Coordinate Geometry
3.	*Powers, Roots	Polynomials	Congruence Transformations
4.	*Rational Numbers	Relations	*Dilatation
5.	Percent	Probability	Displacement
6.	*Applications of Ratio and Measurement		
7.	Lists, Tables		
8.			

GRADE 10 MATHEMATICS (GENERAL)

NUMERICAL METHODS	ALGEBRA	GEOMETRY
Number Applications	*Relations	*Geometric Figures and Properties
Ratio and Rate	*Linear Equations and Inequations	Coordinate Geometry
†††Statistics	Polynomials	Vectors
*Square Roots		
Personal Finance		
Payrolls and Flow Charts		
Applied Trigonometry		
Matrices		

NOTE:

i) *Optional Sections* — [ ] *indicates sections in which all the topics are optional.*

ii) *Optional Topics* — * *indicates sections in which some of the topics are optional.*

iii) *Delayed Implementation* — †† *indicates sections that are to be fully implemented as printed beginning in September 1981.*

††† *indicates sections that are to be fully implemented as printed beginning in September 1982.*

GRADE 9 MATHEMATICS

SECTION	NUMERICAL METHODS	ALGEBRA	GEOMETRY
1.	*The Counting Numbers	*Problem Solving	Basic Notions Of Geometry
2.	Computation with Whole Numbers and Decimals	Introduction to Computing	Constructions
3.	Fractions	The Mathematics of Chance	Transformations
4.	Ratio and Rate		3-D Geometry
5.	Percent		
6.	Data Presentation		
7.	Powers and Roots		
8.	Integers		
9.	*Measurement		

NOTE:

i) Optional Sections [ ] *indicates sections in which all the topics are optional.*

ii) Optional Topics * *indicates sections in which some of the topics are optional.*

GRADE 10 MATHEMATICS

This course is to be based on the topics of Level 3, Grade 9 Mathematics (Basic) developed in the context of relevant applications of the topics

It is suggested that these applications be developed from Themes, such as those outlined in:

i) *Notes for Teachers, Level 3, Grade 10 Mathematics (Basic)*

ii) *Appendix 1, Level 3, Grade 9 Mathematics (Basic)* — Themes

iii) *Appendix 2, Level 3, Grade 9 Mathematics (Basic)* — a sample development of thematic materials

iv) Other resource materials related to Level 3

Other themes may be developed locally.

Themes and/or subthemes used in the Grade 10 course should not duplicate these used to develop the Grade 9 course.

SECTION	COURSE A LEVEL 1 GRADE 9 MATHEMATICS NUMERICAL METHODS	GEOMETRY	COURSE B LEVEL 2 GRADE 9 MATHEMATICS LEVEL 1 GRADE 10 MATHEMATICS NUMERICAL METHODS	GEOMETRY	COURSE C LEVEL 2 GRADE 10 MATHEMATICS LEVEL 1 GRADE 11 MATHEMATICS NUMERICAL METHODS	GEOMETRY
1.	*Non-Computational Use of Numbers	Motion Geometry	Non-Computational Use of Numbers	Motion Geometry	Non-Computational Use of Numbers	*Motion Geometry
2.	*Computational Use of Whole Numbers	*Geometric Constructions	Computational Use of Whole Numbers	*Geometric Constructions	Computational Use of Whole Numbers	*Geometric Constructions
3.	Computational Use of Decimals	Prisms and Pyramids	Computational Use of Decimals	Prisms and Pyramids	Computational Use of Decimals	Prisms and Pyramids
4.	Percent		*Percent		*Percent	
5.	Aids to Solving Mathematical Problems		Aids to Solving Mathematical Problems		Problem Solving	
6.	*Fractions		Fractions		Fractions	
7.	*Data Graphs		*Data Graphs		*Data Graphs	
8.	*Measurement		*Measurement		*Measurement	
9.	Probability		*Probability		*Probability	

NOTE:

i) Optional Sections [ ] *indicates sections in which all the topics are optional.*

ii) Optional Topics * *indicates sections in which some of the topics are optional.*

DIFFERENT LEVELS OF THE PROGRAM

Students entering Grade 7 will be at different levels in terms of their skills and understanding of mathematics. They will also vary in their attitudes toward the subject. This presents a challenge to teachers of Grades 7 and 8 to build programs with enough flexibility to meet the varying needs of their students.

Patterns of class organization vary from school to school and from system to system. Some schools use a complete rotary timetable in Grades 7 and 8; others have some modification of this; still others maintain a one teacher – one class pattern for scheduling time. In all these situations it is vital that the course objectives and expectations be varied enough to accommodate the range of performance levels of students, however broad it may be. It should be noted that the Outlines of Topics do not specify these objectives, expectations, and scope of the program. These are to be determined at the board and/or school level.

The Outlines of Topics for the five levels in Grades 9 and 10 have been developed to meet the needs of the full range of students entering the secondary schools of Ontario.

- The *Level 5 (Advanced)* program is designed for students who have successfully completed Grade 8 Mathematics in the regular program, and who show an interest in studying mathematics to the end of Grade 13 and possibly beyond.
- The *Level 4 (General)* program is designed for students who have completed Grade 8 Mathematics with some difficulty in consolidating the skills and concepts of the program and in understanding abstractions, and/or who do not plan future studies involving mathematics at the university level.
- The *Level 3 (Basic)* program is designed for students who require additional experiences with the concepts and skills of Grades 7 and 8 Mathematics.
- The *Level 2* program is designed for students who have had some exposure to mathematics at least at the Grade 7 level, but whose mathematical understanding is at the Junior Division level, and who are enrolled in special vocational or occupations courses.
- The *Level 1* program is designed for students who have not attempted Grade 7 Mathematics, have a mathematical understanding at the Primary Division or early Junior Division level, and are enrolled in special vocational, occupations, or services courses.

PLANNING THE SCHOOL'S PROGRAM

School Boards, through local curriculum committees and/or school staffs, are responsible for expanding the Outlines of Topics into detailed courses of study for instructional purposes. These locally developed courses should incorporate the policy and shifts in emphasis that are indicated in this revision, while taking into consideration the priorities and resources of the community.

It is not expected that every secondary school will offer courses for each of Levels 1 to 5 in Grades 9 and 10. Local conditions will determine whether this is desirable and/or feasible. It is expected, taking into consideration the student enrollment in a particular grade relative to the staff size and the total subject program of the school, that everything possible will be done to ensure that mathematics courses are available to students consistent with their specific needs, interests, and mathematical abilities.

Specifically, these local programs should provide many opportunities for students to develop confidence in their ability to use mathematics in a wide range of situations. This calls for regular use of examples, investigations, problems, and applications in conjunction with the skills and concepts being developed. These should be related to the needs and interests of the students.

CORE AND OPTIONS

In each of the Outlines of Topics, the topics have been identified as either *Core* or *Optional.* The schools are responsible for ensuring that the locally developed courses include the core topics and that students have opportunities to demonstrate their proficiency with the core program for successful completion of the course.

Many of the core topics review and extend ideas from earlier studies. These topics maintain continuity with the students' earlier experiences and help them to acquire a consistent body of mathematical knowledge that is related to real-world applications. In general, the core topics provide the foundation that is needed for future mathematics courses in the same sequence, and for the study of other subjects. All students will not necessarily need the same detailed development of the core; this provides room for flexibility at the classroom level.

In Grades 7 and 8 and in Levels 4 and 5 of Grade 9 and Level 5 of Grade 10, it has been estimated, by about 30 teachers during the validation stages of an earlier draft of this guideline, that under normal circumstances the core will require approximately 75 percent of the time planned for each course. Their estimate was 60 percent for Level 4 of Grade 10. In these circumstances, the course should be developed as a blend of the core and some optional topics. The timing suggested above does not apply to Levels 1, 2, and 3; this will be discussed later in the document.

In any given course, teachers need not feel restricted to the options given in the Outline of Topics. They may select from other Outlines of Topics or use options of their own choice. Options are useful for enriching the students' understanding of the core topics; previewing topics that are introduced in future courses; introducing valid mathematical topics that are not identified in the program; and providing enrichment and motivation through recreational mathematics.

In classes in which the students are experiencing difficulty with the program, it is advisable to devote more time to consolidating the core topics and less time on the options.

Scope of the Courses

The description of the topics in each outline is deliberately concise and does not indicate a detailed development. The breadth and depth of investigation, the approach or approaches to the topics, and methodologies to be used involve decisions that are to be made at the classroom level. In this way, each course can be better adapted to the particular needs of the students.

Strands and Sections

The Outlines of Topics are organized by *Strands* and

Sections that identify the major topics from which the course is to be developed. The sections contain the particular topics to be studied. The order in which the sections and topics are listed is not intended to indicate the order in which they should be developed. In fact, it is recommended that at least some aspects of the courses be developed by integrating topics from the different strands. There are many linkages between the strands and, in order that mathematics be viewed as a unified whole, these linkages should be used in planning the courses.

SHIFTS IN EMPHASIS

This guideline advocates that the mathematics of the Intermediate Division should be of *practical value* to the students by providing them with the *essential skills and concepts* they will need for life today and in the future. This statement applies to all students in each level. The curriculum should include *concrete experiences* and involve *applications drawn from the students' world.* Real-life problems do not separate themselves into the exclusive domains of arithmetic, algebra, geometry, trigonometry, and so on. Accordingly, this applications-oriented curriculum should provide a *unified view* of mathematics by integrating the development and application of the skills and concepts from the different strands.

Problem solving should be a major focus of the curriculum. It should pervade all topics, rather than be taught as an isolated unit at some time during the year. Skill in problem solving and, particularly, in building *mathematical models* to represent aspects of the real world, is best developed when repeated experiences are spread over time.

The mathematics curriculum should *serve the needs of other subjects,* such as science, geography, physical education, family studies, and technical and commercial subjects. It should *prepare students to live in a consumers' world* – to make wise decisions on personal spending; to plan and carry out home improvements; to judge the merit of products; to interpret and use graphs, charts, diagrams, timetables and other tables, and so on.

While serving the above purposes, the curriculum should also help students to understand and apply the *mathematical principles and skills* that are basic to future mathematical studies. In the Intermediate Division, *the underlying structures of mathematics are not intended to be studied in their abstract forms.* Rather, the structures should be experienced in the specific forms in which they occur from day to day.

The structures may be formalized in the Senior Division, where they can be based on these earlier experiences.

Intensive study and drill on the *manipulative skills of algebra are premature* at this stage. These skills are needed for the development of some topics in the Senior Division, and will be consolidated at that time by students who proceed with advanced or general level studies. Students in the Intermediate Division need more experiences in *building algebraic formulae* as generalizations of arithmetic patterns and of real-life experiences. This is consistent with the increased emphasis to be placed on model building throughout the program.

NEW DIRECTIONS AND NEW TOPICS – A RATIONALE

Numerical Methods

Traditionally, arithmetic has been taught in the elementary schools and virtually ignored in secondary schools and universities in favour of algebra and other major mathematical topics. The research project *Applications of Mathematics, A Nation Wide Survey (1976)* reveals that many of the problems faced by industry, business, and government are being solved by numerical methods making use of computers, programmable calculators, scientific calculators, and simple four-function calculators. This indicates an urgent need for students in the Intermediate Division to expand and apply the number sense and arithmetic concepts and skills learned in the K-6 program. This is particularly true, since the low cost of four-function and scientific calculators and the dropping costs of programmable calculators have brought them within the price range of most people, to the point where almost everyone will own a calculator of some type in the near future – if this is not already the situation today. These people will use calculators to solve their personal problems, whether simple or complex. It is thus important that the curriculum provide more emphasis on numeric approaches for solving problems – whether with a calculator, or without.

The Numerical Methods Strands in the various courses are made up of topics that develop number concepts and skills and apply them in a variety of ways. The study of these topics is intended to help students to develop confidence in their ability to use mathematics in solving real problems, and to become aware of the important place that numbers play in the organization of our lives.

Algebra

The algebra program in the past has been primarily concerned with the manipulation of algebraic expressions into equivalent forms, and to the solution of equations and systems of equations by developing sequences of equivalent, but simpler, equations. Little attention was given to the use of algebraic expressions and equations as models of real life situations.

The revised program is concerned with helping students to build simple algebraic models to represent real-world situations. It is also concerned with numeric substitution in formulae and in polynomials, including polynomials in nested multiplication form, and with solving equations by a variety of techniques including systematic trial. The nested multiplication form provides a simple substitution technique that involves the fewest number of steps and is suitable for use with a four-function calculator without memory. The systematic trial technique for solving equations numerically generalizes to the solution of equations of any order and with "nasty" coefficients – the kinds of equations that describe real-life situations.

Manipulative skills for simplifying algebraic expressions are not being emphasized at this time, since these are not "essential skills needed by everyone today or in the future" and cannot be justified by real life applications. They will be established later in the Senior Division for those students whose program requires their use.

Geometry
The geometry topics at all levels of the program are suited to an experiential development using activities and applications, rather than an axiomatic approach. Again, the process of building mathematical models (in this case, geometric diagrams) to represent physical aspects of the real world should be emphasized at the classroom level. Students should be encouraged to investigate particular cases of geometric properties and to make generalizations based on their intuition. These properties should be related to the real world, wherever possible. This approach requires that students develop skills for constructing accurate diagrams, since properties are to be conjectured from the finished figures. Although deductive reasoning will likely be used informally at various times in the program, it is only at Level 5 of Grade 10 that students are expected to study deductive proofs.

The world is filled with motion, and so motion is of practical significance in a program that is concerned with explaining everyday events. This is one of the reasons why a transformation approach is introduced in the Geometry Strand. This approach extends the study of the physical motions of slides, turns, and flips of the K-6 program to the mathematical models that represent them – translation, rotation, and reflection. Symmetry is of great importance throughout this development.

Expansions and reductions, and distortions, are common in the real world; they are studied mathematically by the topics of dilatation, and stretches and other distortions, respectively. The transformation approach to geometry has value both from a practical and a unifying point of view. For example, reflections are used in the design of optical instruments, such as the right-angled prisms in binoculars. The rotational symmetries of the tetrahedron are evident in the turnstiles at the entrance to subway stations. Dilatations are basic to photographic enlargements and to scale drawings. Translations of lines and angles are common in drafting, and special techniques have been developed for this purpose. The computerized instructions that control a drill press in a modern machine shop cause the work piece to slide and turn in a bewildering sequence of steps. Many artistic patterns are generated by transformations.

All of this provides valid motivation for many students, as once more mathematics is used to explain things of the real world.

The transformation approach has its counterpart in the Level 5 Grade 10 Algebra Strand in the form of mapping rules for the transformations as the basis for developing graphing techniques and properties of functions and relations. It also has its counterpart in the Numerical Methods Strand in the form of matrices that represent transformations. Each of these provide models for the "before and after" aspects of the pre-image and image occurrences in the real world. Matrix representations make it possible to deal with transformation problems using a computer. These topics provide a very powerful example of the unity of mathematics – the same problems are examined by a combination of geometric, algebraic, and numerical methods.

This strand also contains topics on three-dimensional geometry. It is important for the student to develop ways of thinking about and representing the world geometrically and to sharpen his or her spatial perceptions.

Unity in Mathematics
Although the major topics in the program are organized by strands, it is intended that the program be implemented with a unified view that interrelates topics from the different strands and that uses applications as a unifying force drawing these topics together. Planning for this integration of topics should be done by teachers when preparing their courses and their individual lessons. Students should be encouraged to draw from their entire mathematical experience when solving problems and when investigating and explaining situations. Extended problems that involve several mathematical topics could be assigned; teachers might assist students by giving hints and partial answers at several places in the assignment.

Applications
The revised program places a major emphasis on applications of mathematics. For most people this is the prime reason for studying mathematics. Whatever its nature, the mathematics program has failed if students are unable to relate their studies to the real problems they face now, or will face in the future. Everyone uses mathematics, in some form or other, in everyday life. This curriculum is intended to help students to recognize these occurrences, to develop an understanding of them, and to extend this understanding to situations that are presently outside their comprehension. This requires that students learn the skills that are essential for handling real-life applications. Through this kind of learning, student understanding of mathematical principles should improve. Those who plan further studies through the Senior Division, and beyond, will have a secure base on which to build more sophisticated ideas and applications.

Applications, problem posing, and problem solving should be interspersed throughout the program, rather than being developed as isolated topics. In a similar way, students can learn mathematics more successfully if they apply their mathematical knowledge in a wide range of subjects. Wherever possible, coordination of the mathematics program with other programs, such as science or geography, will pay dividends. At the same time, even though the development of English in the formal sense rests primarily with teachers of English, mathematics teachers must share the responsibility with all other teachers for maintaining good standards of both oral and written expression.

The present revision incorporates shifts in emphasis that are long overdue. It has retained those topics and concepts from earlier curricula that have an orientation toward real-life experiences and that are essential for everyone. It introduces topics and approaches that are new to the Intermediate Division curriculum such as statistics, probability, flow charts, model building, solving equations by systematic trial, matrices, transformation approaches, displacements, and applied trigonometry. Some of these are optional.

Even though the topics in this revision support the emphasis on applications and problem solving, the responsibility for seeing that these components become a reality rests with the teachers. They will need to "think applications" and to encourage their students to

look at the environment from an applications viewpoint. Teachers should not overlook the valuable ally they have in calculators. They extend the breadth and depth of applications that can be explored by providing a strong motivational force and by removing the drudgery of tedious calculations and keeping logical thought processes flowing.

This shift toward practical applications applies to courses in all grades and levels of the Intermediate Division. The need for a curriculum suited to life today and in the future is paramount for all students regardless of the level of difficulty of the courses they are taking. However, classroom implementation strategies and activities must vary for the different levels in the program and for students of different mathematical abilities.

Local curriculum committees could spend their time profitably by identifying relevant applications of mathematics in the local community and then developing related classroom materials at various levels of difficulty for use throughout the system. In turn, these materials could be exchanged with other jurisdictions who have similar projects.

When searching for applications, schools in the north might consider materials associated with hard rock mining, forestry, fishing, and natural resources. In the south-west, the furniture industry provides examples of factory flow charting, costing, and geometrical design. The agricultural pursuits in the south provide examples of cumulative frequency curves related to the growing season, such as for corn heat units. In this same region, many examples of area calculations related to field sizes, crop densities, greenhouse usage, and so on, are found. The canneries provide examples of the stacking of differently shaped solids inside containers. Virtually every part of Ontario has some local industry, commercial enterprise, or other activity which contains worth-while mathematics, often in heavily disguised form.

The mathematics of everyday life is primarily concerned with simple business transactions, ranging from the purchase of food and clothing to negotiating loans and mortgages. These give ample opportunity for the use and development of number skills, at ever-increasing levels of sophistication as the student moves through the Intermediate Division. Consumer mathematics properly incorporates the measurements, estimates, and evaluations which precede any purchase. Such simple activities as the measurement of windows, floor areas, and dress patterns are essential to a well-run household. An accurate list of materials, combined with comparative pricing, can effect considerable savings on home improvement projects of all kinds. Students, at some stage in the program, should examine some of the implications of money flow, such as the tax base and expenditures of the local community. In a similar way, it is possible to incorporate accident statistics, inflation, and car insurance rates into a single project which is largely numerical in nature. Students might examine legal and medical fee structures, and legal aid and medical plans from a numerical viewpoint. The magnitudes of the sums involved are of considerable social and mathematical interest.

The normal activities around and within the home provide many examples of mathematics which are of life-long utility. Spreading grass seed or fertilizer involves ratios, areas, and concentrations; the indoor gardener faces essentially the same problem on a smaller scale when feeding plants. Nearly every recipe, from making porridge to home canning, involves simple computations to increase or decrease the amounts called for in the list of ingredients. Counting calories is another activity that involves ratio and proportion; this could lead to the more exacting problems faced by a hospital dietician.

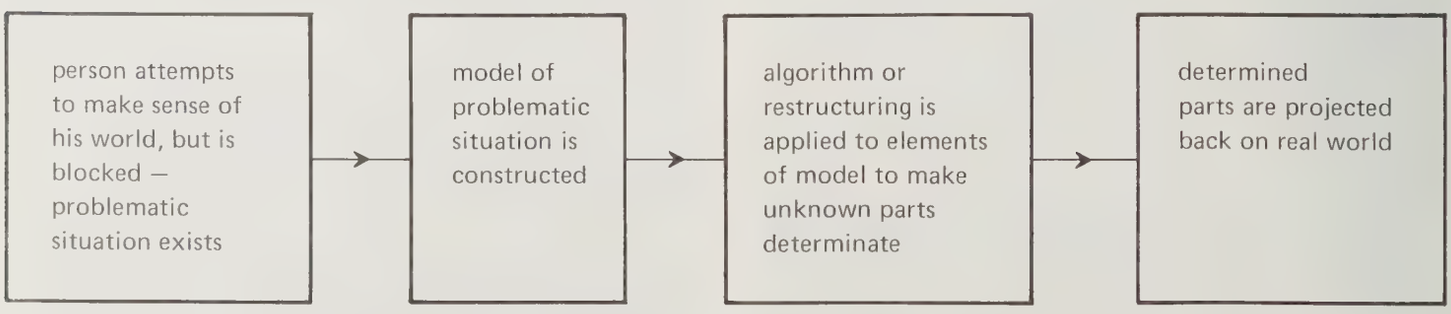
The core part of the Intermediate Division program provides numerous opportunities to use numbers in real-life situations. From these experiences a subconscious awareness of the structures of mathematics will be nurtured; they only need to be highlighted by the teacher in order to identify the essential mathematical concepts that are involved.

Applications form unifying links for a variety of topics throughout the program and are expected to be an integral part of locally produced programs.

It must be appreciated that inappropriate applications can detract from, rather than enhance, a mathematics program. For this reason applications should, if at all possible, be based upon local conditions and activities, and also be presented at a level of difficulty which the students can comprehend.

Problem Solving

The ability to solve problems is the ultimate, but elusive, goal of education. In it, past experiences are drawn upon – sometimes in a systematic manner, but often in flashes of creativity and intuition. It does not belong exclusively as a part of mathematics; it crosses the thresholds of all subjects, and reaches into everyday life. Much that is viewed as applications may be classified as problem solving, but not all. Certainly not all problem solvings can be classified as applications. An interesting schema* to represent the steps in problem solving is given below.

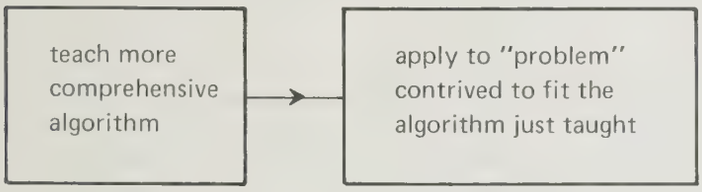


*Floyd Robinson, O.I.S.E., Letter to the Intermediate Division Mathematics Committee, February 1977.

Although the schema applies to more general problem solving situations, in this case it is being considered in relation to mathematics. To solve problems in a systematic manner it assumes there will be:

- development of the students' ability to identify and formulate problems of a logical – quantitative nature
- coherent extension of model building skills
- widening awareness of the genuine applications of mathematical ideas in both school subjects and real life, as well as an understanding of the uniqueness and limitations of logical thought
- use of increasingly sophisticated models and procedures to deal with problems arising from real-life experience.

Traditional mathematics programs from the past may be represented by the following schema:



The two schemata illustrate the differences between the approach proposed by this guideline and that used in traditional programs. This revision is committed to placing more emphasis on applications and on model building. These elements, when implemented effectively into the school program, will produce students who are better prepared to tackle problem solving in a systematic manner. This requires teachers to acknowledge the importance of the skills of problem solving and to give them the priority they deserve. Increasing the emphasis on applications and developing skills of model building represent a challenge to develop a mathematics program that is relevant to the modern world.

Model Building

The ability of mathematics to simulate many aspects of reality is one of its great powers. This is done by building models to represent real situations and by "playing" with the models using rules that stimulate the nature of the real things. When the model is altered, it sometimes reveals information about the real world that was not known before the simulation activity began. Thus models play an extremely important role in the worlds of science, research, business, technology, government, and so on.

Through well-designed models, man is able to predict the future with uncanny accuracy. The success of many operations related to the above areas of endeavour is strongly influenced by predictions that are made from the models. For example, models of the pollutants entering Lake Erie are invaluable in predicting long term effects and possible remedies. Such models can be manipulated with comparative ease, whereas it is enormously difficult and expensive to alter actual physical situations during long term environmental experiments.

There is a great need today for people skilled in creating models. The first stage in developing these skills can be established by working with simple situations. These may relate directly to established models, such as formulae for area, or to less well defined sociometric problems. For example, students might develop a model that predicts what would happen if a new housing estate with "h" houses was developed in the school district. What happens to the school staff and facilities? Is there any effect on local business? Is the school football team likely to be more successful? It is in the effort to quantify these relationships that students learn a great deal about mathematics, economics, and social interaction.

There are numerous classes of mathematical models; directed graphs, route matrices, Monte Carlo simulations, and so on. However, in the Intermediate Division the emphasis should be on algebraic and geometric models. Although algebraic models are not new to this division they have usually been presented in a completed form, such as formulae. This revision places greater emphasis on the development of algebraic models, and less on the manipulation of algebraic expressions for its own sake.

Physical situations are often represented by models in the form of accurately drawn diagrams. For example, the floor plan of the school, a pattern of a dress, the architectural plans of a house, and so on are geometric models of the real things that in most cases are still on "the drawing board". The geometry program should be developed using real-life experiences as the source of problems requiring geometric models.

Calculators

It is inevitable that a program which deals with realistic situations will also have to cope with the rather "unpleasant" numbers that are ever present in the real world. Students will face problems which do not have numerical simplicity, and in this respect calculators will help to simplify tedious, but non-trivial, calculations en route to the final solution. The mini-calculator lends itself to discovery exercises; and also to the checking of drill-type arithmetic in a non-threatening way.

The rapid expansion of numerical methods in recent years hinges upon the availability of calculators. By removing much of the arithmetic drudgery from calculations, calculators have increased the time available for logical mental processes which are essential to mathematics. In addition the calculator has opened alternative methods for solving complex problems, and provides potential for adding new insights to instructional processes.

There is a need for informal research by teachers at the classroom level to establish and evaluate ways of using the mini-calculator to investigate many of the traditional topics in the curriculum such as terminating and non-terminating decimals, operations with integers, powers with integral exponents, compound interest, approximation errors resulting from truncation, functions and their inverses, addition of fractions, percentage – the list could go on. At this stage it would seem that the calculator, when properly used, could provide motivation and new insight to these topics – but only with experience at the classroom level can we determine its long range effects.

Care must be taken to ensure that students do not rely exclusively on calculators. There is a need to include in the program a variety of experiences that require mental arithmetic and simple pencil and paper calcula-

tions. There is no question that the human brain can outperform the machine on simple calculations. What has yet to be determined, and probably so for each individual, is where the fine line is positioned between what the brain can do best and where the calculator should take over. Certainly, part of the classroom strategy should be to structure situations in which the alert mind can outperform the machine. Situations abound where the mind can't.

Students should also appreciate the flow-diagram capabilities of the mini-calculator, and be encouraged to develop efficient algorithms for their use. These activities make a fitting prelude to computer programming.

Statistics

Statistics is becoming increasingly important in our complex world. However this subject is still new in the elementary and secondary programs and must be treated with care in the mathematics program. When developing local programs from these outlines, it should be kept in mind that statistics should be seen by the students as being useful.

Initially the emphasis should be on the representation of statistical data using various methods. These visual representations should be seen as useful techniques for compressing large amounts of data into comprehensible forms. The second major emphasis should be on inferential statistics; that is, deductions and inferences which can be made from samples. The ideas of sample size, reliability of inference, and so on should be approached intuitively and experimentally at this stage. Measures of central tendency – mean, median, and mode – are only of significance in relationship to the previous comments, and should not be computed for their own sake.

Curriculum committees that write materials for statistics should bear in mind the distinction between statistics and probability. Statistics is founded on a data base which is collected or measured. Probability is a mathematical model which, in some cases, produces results which are close to the results of statistics.

Matrices

Lists and tables of numbers are used in a variety of situations to summarize data. Today, no person can avoid using numbers in this form and, in many instances, operating with these arrays. Sports data, such as team standings, are expressed in tabular form; these tables are updated daily by addition of the tables. Stock taking can be summarized in this form, as can the value of each stock item, with the value of the stock determined by multiplication of the arrays. Shopping lists, bus timetables, and conversion tables are other examples of uses of matrices that are suitable for the Level 5 Grade 9 program. The formalization of a matrix as a mathematical entity and the structural view of this branch of mathematics should be reserved for later in the Senior Division.

Unquestionably matrices provide a very concise way of representing a great deal of information, be it the practical examples as above or the more mathematical uses, such as the array of coefficients of a system of n equations in n variables, or the matrix for a rotation of 90° counter-clockwise about a point.

Since they are arrays of numbers, matrices can be stored in a computer and operated on with great ease. For this reason, matrices are one of the most powerful mathematical tools in the field of applied mathematics. They are used in many disciplines other than mathematics such as medicine and economics. There are few students who go on to studies beyond secondary school who will not encounter matrices in their work. Most citizens will need to read data presented in tables or lists.

Matrices, like transformations, provide unity in modern mathematics as they draw together ideas from geometry and algebra and deal with them by numerical methods. The integration of the strands is strengthened in this way.

It is for the above reasons that the opportunity to introduce matrices informally is given in Grades 9 and 10 of the Level 5 program.

Experiential Approaches

Experiential approaches are fundamental to the implementation strategies of this program. Students of today and citizens of the future should be able to look at the world they know and see it with expanded understanding without being handicapped by a lack of applicable mathematical knowledge. When appropriate, every person should be able to recall and apply mathematical ideas from their past experiences and to reason mathematically when dealing with problems. Thus, the program should evolve from a variety of practical experiences with mathematics at all grades and at all levels of difficulty. The program should make use of concrete materials, investigations, "hands-on" experiences, and emphasize real-world situations that can be imitated by classroom activities. This approach must be complemented by the development of the skills and concepts that are needed to deal with the real world. The Outlines of Topics, for all levels of difficulty, identify the ideas that are to be developed in experiential ways. They also provide the foundations that are needed for the Senior Division program and the preparation of students for post-secondary education, as well as for day-to-day living.

THE PROGRAM FOR GRADES 7 AND 8

The school's mathematics program for Grades 7 and 8 is to consist of courses that are to be developed locally from the Outlines of Topics, as provided on pages 17-23. The general ground rules for this development are discussed in the section entitled Core and Options found on page 9. These courses will provide a consolidation, refinement, and extension of ideas developed in the K – 6 program based on The Formative Years. They are to be consistent with the policy and directions of this guideline and should provide sufficient flexibility to meet the needs of a wide range of students.

The major topics on which the courses are to be built are summarized in the chart below. See the Outlines of Topics for the detailed description.

GRADE 7 MATHEMATICS				GRADE 8 MATHEMATICS		
SECTION	NUMERICAL METHODS	ALGEBRA	GEOMETRY	NUMERICAL METHODS	ALGEBRA	GEOMETRY
1.	Data Graphs	Communications	*Basic Notions in Geometry	*Number Applications	*Formulae and Equations	Accurate Constructions
2.	Number Applications	Equations	*Accurate Constructions	*Fractions, Ratio, Percent	Relations	*Congruence †Transformations
3.	*Percent	Relations	Patterns with tiles	*Data Graphs †	Probability	*Properties of Plane Figures
4.	*Fractions, Ratios		*Transformations	Integers	Flow Charts	*Dilatation †
5.	*Factors		*Enlargements and Reductions	Rational Numbers in Decimal Form		*3-D Geometry
6.	Integers		*3-D Geometry	*Measurement of length, area, volume		
7.	Other Arithmetics			*The circle and related measures		
8.	*Measurement of length, area, volume					

NOTE:

i) Optional Sections — ⋯ *indicates sections in which all the topics are optional.*

ii) Optional Topics — * *indicates sections in which some of the topics are optional.*

iii) Delayed Implementation — † *indicates sections that are to be fully implemented as printed beginning in September 1980.*

This chart indicates that most of the major topics in Grade 7 overlap the K – 6 mathematics program. This has been done deliberately. Programs based on The Formative Years have only been in effect since 1975. It will be at least 1981 before students entering Grade 7 will have had their entire school experience based on The Formative Years. In the interim it may be necessary to devote more time to the core topics of the Grade 7 and Grade 8 courses. As the policies in The Formative Years become more fully implemented, it will be possible to incorporate more of the optional topics.

Because growth in mathematical understanding should be continuous and coherent, it is vital that teachers of pupils of different grades share the responsibility for developing courses. This is relatively easy to organize in K to 8 schools where students from three divisions are housed in the same building. When teachers are situated in two or even more locations, it requires a greater commitment to arrange the time needed for continuing dialogue. In any event, joint planning is essential if the goals of the program are to be achieved.

In Grade 8, the sections Data Graphs, Congruence Transformations, and Dilatation are coded for delayed implementation as printed until September 1980. These topics are new to the Intermediate Division program in Ontario, and they are developed sequentially within the revision. They cannot be effectively implemented as printed in Grade 8 until the incoming students have the necessary background from Grade 7. This delayed implementation is not intended to cause a vacuum in the program in 1979-80. Teachers of Grade 8 are expected to teach modified units based on these sections by using fewer of the topics within each section and including some of the Grade 7 ideas coupled with their Grade 8 extensions.

It is recognized that different students entering Grade 7 have different levels of mathematical knowledge and understanding. This diversity requires that courses of different levels of difficulty be planned – all dealing with the same core topics, but with different options, scope, and expectations. The breadth and depth of treatment, approaches, methodologies, examples and applications, and situations being investigated should all be modified to suit the nature of the students in the specific classes. This planning could be done for the school system by a local curriculum committee, or in the school itself.

Individual teachers should learn the levels of development of the incoming students, so that the "day in – day out" classroom experiences can be planned more effectively. In the best interests of the students and to assist subsequent teachers in building their courses, records should be kept of the content and scope of the modified courses and of the achievement of individual students.

It is important that all students receive a variety of experiences throughout the year that are designed to consolidate and extend their knowledge of numbers, their number sense, and their ability to use numbers in a variety of applications and problem-solving situations. In particular, the Numerical Methods Strand in each course is intended to permeate the course – both in the planning stage and in the implementation. Many of the sections in the other strands have a number applications component in them. A judicious use of games, puzzles, and other activities can do much to motivate students while extending their sense of number and consolidating their skills in subtle ways. The most significant change in the program occurs in the Geometry Strand. This change extends the geometry topics of The Formative Years and places major emphasis on experiential approaches.

The Outlines of Topics for Grades 7 and 8 are provided on the following pages.

MINISTRY OF EDUCATION

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DRAFT COPY

GRADE 7 MATHEMATICS

OUTLINE OF TOPICS

NOTE 1. *This is a DRAFT COPY for validation purposes and for optional implementation during the school years 1977 – 1978 and 1978 – 1979. Reactions should be forwarded to the Intermediate Mathematics Coordinator, Ministry of Education, Curriculum Branch, Mowat Block, Queen's Park, Toronto M7A 1L2.*

NOTE 2. *In designing courses at the local level, topics within the dashed rectangles [ ] are to considered optional; all other topics are core.*

September 1977

NUMERICAL METHODS (N)

1. DATA GRAPHS

a) Reading and interpreting pictographs and bar graphs

b) Collecting data, tally charts

c) Representing whole number data by pictographs and bar graphs

2. NUMBER APPLICATIONS

a) Place value

b) Operations with whole numbers and decimals; estimating answers; applications in real life situations

c) Practice in computation through games, puzzles, and activities

d) Rounding; applications in real life situations

e) Multiplication and division with powers of ten

f) Properties of arithmetic operations, and the roles of 0 and 1 identified in b) and c)

g) Use of these properties in simplifying numerical expressions

3. PERCENT

a) Percents as hundredths

b) Conversion of percents to decimals, and of decimals to percents

c) Applications to sales tax, simple interest, discount

d) Applications to gain or loss

ALGEBRA (A)

1. COMMUNICATIONS

a) Use of parentheses

b) Use of diagrams to represent mathematical situations

c) Use of flow charts in numerical and non-numerical problems

2. EQUATIONS

a) Converting word statements to mathematical notation; concept of variable

b) Solving equations in one variable by systematic trial, using whole numbers, decimals, and fractions

3. RELATIONS

a) Representing non-numerical and numerical relationships by arrow diagrams; mappings

b) Representing relations using ordered pairs on a grid

GEOMETRY (G)

1. BASIC NOTIONS IN GEOMETRY

a) Use of geometric terms in the correct context

b) Intersecting, perpendicular, and parallel lines; properties

c) Symmetries of plane figures; corresponding parts

d) Classification of angles, triangles, and quadrilaterals

e) Polygons; symmetry properties of regular polygons

2. ACCURATE CONSTRUCTIONS

a) Constructing congruent segments, congruent angles, perpendicular lines, perpendicular bisector, angle bisector, and parallel lines

b) Constructing rectilinear figures

c) Measuring and constructing angles with a protractor; angle properties of various figures

3. PATTERNS WITH TILES

a) Fitting two or more congruent tiles; properties of the figures formed

b) Relating turns, flips, and slides to the figures in a)

c) Tangram and polyomino puzzles; the symmetries of the tiles and of the figures

d) Tiling the plane

e) Identifying congruent figures, similar figures, line and angle properties in tiling patterns

f) Identifying slides, turns, flips and symmetries in tiling patterns

g) Area concepts in tiling patterns

NUMERICAL METHODS (N)

4. FRACTIONS, RATIOS

a) Concept of a fraction

b) Operations with fractions, reciprocals; practice; applications to real life problems

c) Ratios as comparisons

d) Equivalent ratios; applications in practical problems

e) Rate, developed from real world examples

5. FACTORS

a) Composite numbers in factored forms, and in prime factored form

b) Powers; evaluation of powers

c) Divisibility tests for numbers such as 2, 3, 4, 5, and 9

d) Lowest common multiple

6. INTEGERS

a) Practical examples that establish the need for integers

b) Locating integers on the number line; order

c) Addition and subtraction with integers

7. OTHER ARITHMETICS

a) Real life modular arithmetics: 12 and 24 hour clocks; week; month; year; 360° in a complete rotation

b) Numeration bases developed through activities with concrete materials

c) Addition and subtraction using bases less than ten

ALGEBRA (A)

GEOMETRY (G)

4. TRANSFORMATIONS

a) Image of a figure under a slide, turn, or flip

b) Translation, rotation, and reflection related to the motion in a); the fundamental property of each

c) Constructing translation, rotation, and reflection images; congruence of corresponding parts

d) Line-symmetry and rotational symmetry as mappings; congruence of corresponding parts

e) Given a figure on a squared grid, drawing its image on a distorted grid

5. ENLARGEMENTS AND REDUCTIONS

a) Enlarging and reducing geometric figures using geoboards, dot paper, tiles, or grids; shape, size, other properties, similar figures

b) Scale drawings representing real life situations; scale ratio; converting scale measurements to actual measurements; maps

c) Similar solids

6. THREE-DIMENSIONAL GEOMETRY

a) Identifying real world objects as solids, shells, or skeletons

b) Polyhedra in the real world; constructing regular polyhedra from their nets

c) Constructing skeletons of pyramids and prisms

d) Sketching 3-D objects from different perspectives

e) Identifying planes of symmetry of 3-D objects

NUMERICAL METHODS (N) | ALGEBRA (A) | GEOMETRY (G)

8. MEASUREMENT OF LENGTH, AREA, VOLUME

a) Estimating and measuring the length of line segments and curves, and the perimeter of figures

b) Formulae for the perimeter of rectilinear figures generalized from numerical cases

c) Estimating and measuring areas of regular and irregular figures

d) Formulae for the area of a square, rectangle, triangle, and parallelogram generalized from numerical cases

e) Volumes of rectangular solids developed through activities; calculation of volumes

f) Capacity of a container

g) Applications of perimeter and area using measurement and/or calculation

h) Measurement of the volume of irregular solids using liquid displacement

i) Estimating volumes of rectangular solids, and capacities of containers

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GRADE 8 MATHEMATICS

OUTLINE OF TOPICS

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NOTE 2. *In designing courses at the local level, topics within the dashed rectangles ⌈ ⌋ are to be considered optional; all other topics are core. Sections coded by † are to be fully implemented as printed beginning in September 1980.*

September 1977

NUMERICAL METHODS (N)

1. NUMBER APPLICATIONS

a) Practice with whole numbers and decimals; applications in real life situations

b) Evaluation of powers with whole number exponents; applications

c) Numerical development of the rules for multiplication and division with powers

d) Geometric significance of square and cube

e) Roots of perfect squares

f) Using calculators in above topics; decimal factors

2. FRACTIONS, RATIO, PERCENT

a) Computation with fractions; practice; applications in real life situations

b) Interpreting scale diagrams by measurement and/or calculation

c) Applying ratio and rate in real life situations

d) Applying percent in problems related to sales tax and simple interest, and to other topics such as discount, gain or loss, commission, inflation

e) Extended problems involving percent

ALGEBRA (A)

1. FORMULAE AND EQUATIONS

a) Converting word statements to mathematical notation; concept of variable

b) Developing formulae by generalizing from numeric cases

c) Evaluating a variable in formulae and algebraic expressions by substituting numerical values

d) Business formulae related to situations such as interest and commission

e) Solving equations in one variable by systematic trial

2. RELATIONS

a) Non-numerical and numerical relations; arrow diagrams; mappings

b) Representing relations using ordered pairs on a grid

c) Linear relations

GEOMETRY (G)

1. ACCURATE CONSTRUCTIONS

a) Constructing congruent segments and angles, perpendicular lines, parallel lines, perpendicular bisector, angle bisector; tests for parallel and perpendicular lines

b) Constructing a triangle congruent to a given triangle; sufficiency conditions; constructing special cases of triangles and quadrilaterals

c) Constructing a circle; locating the centre of a circle and of a circular arc; concentric circles

2. † CONGRUENCE TRANSFORMATIONS

a) Constructing translation, rotation, and reflection images from the fundamental properties; other properties

b) Reflection, translation, and rotation images in the real world

c) Tests to show whether two congruent figures are related by reflection, translation, or rotation; constructing the reflection-line, translation arrow, or rotation centre and angle

d) Reflection images in curved mirrors and flat mirrors; "reflection in a line" as a model of reflection in a flat mirror

NUMERICAL METHODS (N)

3. † DATA GRAPHS

a) Constructing and interpreting bar and line graphs

b) Population samples with simple attributes expressed in ratio form

c) Extending the tally chart to include percentage distribution

d) Representing percent distributions by circle graphs

e) Data distribution expressed as fractions with a sum of 1

f) The mode of the population; modal frequency represented as a percentage of the population

4. INTEGERS

a) Practical examples that establish the need for integers

b) Locating integers on the number line; order

c) Addition and subtraction with integers; practice

d) Multiplication and division with integers; practice

5. RATIONAL NUMBERS IN DECIMAL FORM

a) Converting fractions to decimals

b) Demonstrating that the decimal form of rationals either terminates or repeats

c) Investigating repeating number patterns with a calculator

d) Rounding errors resulting from successive calculations, in particular with calculators

e) Representing large and small numbers in scientific notation

ALGEBRA (A)

3. PROBABILITY

a) Listing the possible outcomes of simple experiments

b) Distinguishing the number of required outcomes from the number of possible outcomes

c) Formula for probability

d) Comparison of predicted and experimental results

e) Using sets to represent different attributes within the population

f) Using Venn diagrams to represent and analyse the situations in e)

4. FLOW CHARTS

a) Representing sequential activities by flow charts

b) Arranging disordered events into logical order

c) Using a flow chart to form an equation such as $3x - 2 = 11$; solution by inverse flow chart

d) Flow charts of algorithms for calculators

GEOMETRY (G)

3. PROPERTIES OF PLANE FIGURES

a) Properties of triangles, parallel lines, and quadrilaterals; the Pythagorean Theorem

b) Applying the properties of a) in numerical exercises

c) Rigidity and non-rigidity of polygonal figures; real life applications

d) Polygons; properties of regular polygons; symmetry

4. † DILATATION

a) Enlarging and reducing figures using grids

b) Drawing dilatation images, scale factor; properties

c) Applications of enlargements and reductions in the real world; scale drawings; maps

d) Tiling patterns to illustrate properties of dilatations

5. THREE-DIMENSIONAL GEOMETRY

a) Constructing skeletons of pyramids and prisms

b) Sketching 3-D objects from different perspectives

c) Making polyhedra from their nets; stacking solids in space

d) Symmetry of 3-D objects; relationship of planes, axes, and point of symmetry; real world applications

e) Dilatation images of 3-D objects

f) Reflections in parallel mirrors, in intersecting mirrors; the image in the right angle corner of two intersecting mirrors

NUMERICAL METHODS (N)	ALGEBRA (A)	GEOMETRY (G)

6. MEASUREMENT OF LENGTH, AREA, VOLUME

a) Formulae for the perimeter and area of rectilinear figures generalized from numerical cases

b) Applications of perimeter and area using measurement and/or calculation

c) Surface area of rectangular solids and prisms

d) Development of formulae for the volume of rectangular solids and prisms; applications

e) Estimating volumes of rectangular solids, and capacities of containers

7. THE CIRCLE AND RELATED MEASURES

a) Investigating circular objects to find circumference/diameter, and area/(radius)^2 as an approximation of π

b) The surface area of a cylinder related to its net

c) The volume of a cylinder as (area of base) (height)

NOTE: † indicates sections that are to be fully implemented as printed beginning in September 1980.

THE PROGRAM FOR LEVEL 5 (ADVANCED)

The Level 5 program is designed for students who

- have successfully completed Grade 8 mathematics
- plan to study mathematics at least to the end of Grade 12 in Foundations of Mathematics courses, but possibly to Grade 13 or beyond
- express interest in university studies or other post-secondary education in which mathematics will play at least a service role

The school's program is to consist of courses that are developed locally from the Outlines of Topics for Level 5 as provided on pages 26-33. The general ground rules for this development are discussed in the section entitled Core and Options found on page 9. These courses are designed as extensions of the K-8 program of the feeder elementary schools, and provide a firm basis for students who may proceed to study mathematics in Grade 13. As with all courses in the Intermediate Division, it is intended that they evolve from practical experiences and activities with concrete materials. A strong intuitive base should be established first, leading to generalizations and abstractions that are appropriate to the students' mathematical maturity.

Although these courses are described as advanced level, there will likely be a wide range of abilities and needs within each class. Subsequently, the school should build sufficient flexibility into the courses to meet the needs of gifted students at the one extreme, and of students who are experiencing some difficulty in comprehending the material at the other.

The chart below summarizes the major topics on which courses are to be developed.

LEVEL 5 (ADVANCED)

	GRADE 9 MATHEMATICS			GRADE 10 MATHEMATICS		
SECTION	NUMERICAL METHODS	ALGEBRA	GEOMETRY	NUMERICAL METHODS	ALGEBRA	GEOMETRY
1.	Number Applications	*Flow Charts ††	*Circle Applications	Radicals	*Equations and inequations	*Coordinate Geometry
2.	*Statistics ††	Equations	*Coordinate ††Geometry	Applied Trigonometry	Relations	*Coordinates and †††Transformations
3.	*Powers, Roots	*Polynomials	*Congruence ††Transformations	*Statistics †††	Probability	*Isometries †††
4.	*Rational Numbers	*Relations	Dilatation	Matrices	Computers	*Deductive †††Geometry
5.	*Applications of ††Percent, Ratio, Measurement	Probability	Displacement		*Polynomials	Vectors
6.	Matrices					*3-D Geometry

NOTE:

i) Optional Sections — [ ] *indicates sections in which all the topics are optional.*

ii) Optional Topics — * *indicates sections in which some of the topics are optional.*

iii) Delayed Implementation — †† *indicates sections that are to be fully implemented as printed beginning in September 1981.*
††† *indicates sections that are to be fully implemented as printed beginning in September 1982.*

To meet the needs of all students taking Level 5 courses in either Grade 9 or Grade 10, the school may plan variations on each Outline of Topics. Although these courses must deal with the same core topics, they can differ in depth and breadth of treatment, approaches, methodologies, activities, applications, selection of options, performance levels expected, and evaluation procedures. It is important that all students gain a sense of achievement and experience success in their endeavours.

With this diversity in the courses offered, it is important for the school to keep an accurate record of the nature of the courses, the students taking them, as well as the performance levels of the students. In this way, unnecessary repetition of topics or gaps in topics can be minimized in successive courses.

The nature and content of the courses offered in Grades 9 and 10 will, by necessity, evolve during the first years of implementation of this guideline. As discussed in the previous section for Grades 7 and 8, the complete K – 8 program will not be fully established immediately. This in turn will affect the nature of the program that is possible in Grades 9 and 10 during the next few years, with the core topics requiring more time during this period.

The delayed implementation of some sections as printed for Grade 9 until 1981 and for Grade 10 until 1982 will help with the introduction of the new program. These sections are both new and sequential in nature; thus, in most situations, their full implementation will need to be delayed. It is intended that some ideas will be developed on these sections in the interim period, probably based on fewer topics in which the development will involve background ideas from earlier grades.

It is important for secondary school teachers to interact with teachers of the feeder elementary schools, in order that the program will flow smoothly from Grade 8 to Grade 9 and evolve from year to year toward full

implementation of all topics by 1982-83. Secondary school teachers should know the characteristics of incoming students in order to assist them with their course selection.

The overview in the chart on the previous page indicates that an even balance of topics from each of the strands may be developed in Grades 9 and 10. Since numerical methods are now being extended into the secondary schools, teachers should consciously apply these methods to topics of the Algebra and Geometry Strands. Flow charts, numerical solutions of equations, evaluation of algebraic expressions and formulae, probability, circle applications, computers, calculators, coordinate geometry, graphing, dilatation, displacements, coordinates and transformations, mappings, vectors, 3-D geometry, matrix representation of transformations, and applications of mathematics all have aspects that can be developed using numerical approaches.

The real world is strongly influenced by calculators and computers of all degrees of sophistication; they are becoming more and more a part of everyone's experience. It is vital, therefore, for the school to ensure that every student increases his or her number sense and confidence in working efficiently with numbers to solve problems. Calculators are so much a part of the world today that it is crucial for students to learn their full capabilities and the dangers inherent in their misuse. At the same time, the program must recognize that there is still a need for mental arithmetic and simple pencil and paper calculations. The human mind is still the world's most valued "machine", because it is capable of decision-making and logical thought. The calculator is programmed by this mind, and is essentially an appendage to it. It extends man's potential to perform calculations with large numbers in routine ways and to deal with problems and applications otherwise beyond human capabilities.

MINISTRY OF EDUCATION

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LEVEL 5 GRADE 9 MATHEMATICS (ADVANCED)

OUTLINE OF TOPICS

NOTE 1. *This is a DRAFT COPY for validation purposes and for optional implementation during the school years 1977 – 1978 and 1978 – 1979. Reactions should be forwarded to the Intermediate Mathematics Coordinator, Ministry of Education, Curriculum Branch, Mowat Block, Queen's Park, Toronto M7A 1L2.*

NOTE 2. *In designing courses at the local level, topics within the dashed rectangles [_ _ _ _ _] are to be considered optional; all other topics are core. Sections coded by †† are to be fully implemented as printed beginning in September 1981.*

September 1977

NUMERICAL METHODS (N)

1. NUMBER APPLICATIONS

a) Practising computational skills with whole numbers, decimals, percents, and integers; applications in real life situations

2. †† STATISTICS

a) Data gathering methods such as personal interviews, questionnaires, and measurement activities

b) Random, clustered, and stratified sampling

c) Practical experience in data surveying

d) Organizing data into frequency tables

e) Representing data by graphs and other methods

f) Determining the mean, median, and mode

g) Appropriateness of mean, median, and mode as measures of central tendency

h) Techniques to simplify the calculation of mean and of median.

i) Non-rigorous predictions based upon samples

3. POWERS, ROOTS

a) Powers with integral exponents, their evaluation

b) Laws for multiplication and division with powers; numerical and algebraic cases

c) Scientific notation; applications to problems in other disciplines

d) Decimal approximations of square roots; applications in real life situations; Pythagorean Theorem

e) Newton's method for calculating square roots

f) Powers with fractional exponents, and decimal exponents

ALGEBRA (A)

1. †† FLOW CHARTS

a) Flow charting as an organizational procedure

b) Loops for repetitive sequences of operations; applications to solving problems and generating sequences

c) Efficient algorithms for use with calculators

2. EQUATIONS

a) Solving linear equations in one variable by systematic trial, and by formal method

b) Solving word problems

c) Manipulating formulae; applications

d) Developing formulae from diagrams, numerical problems, and measurement activities

3. POLYNOMIALS

a) Adding and subtracting polynomials

b) Product of a monomial and a polynomial

c) Common factors

d) Polynomials in nested form

e) Solving non-linear equations in one variable by systematic trial, using nested form

f) Product of two binomials

g) Factoring trinomials of the form $x^2 + bx + c$; difference of squares

GEOMETRY (G)

1. CIRCLE APPLICATIONS

a) Basic constructions related to the fundamental property of a circle and to line-symmetry; point on a perpendicular bisector; point on an angle bisector

b) Properties related to the circle; applications in numerical cases

c) Concurrent lines of a triangle; circumcircle and incircle; centre of gravity

2. †† COORDINATE GEOMETRY

a) Drawing figures on a grid given the coordinates of their vertices

b) Images of the figures in a) under reflection, translation, rotation, and dilatation

c) Slope of a segment in numerical cases; applications in the real world

d) Distance between points in numerical cases using the Pythagorean Theorem; applications in the real world

e) Properties of the image of a segment under translation, dilatation, and special cases of reflection and rotation

f) Images of rectilinear figures under vertical and horizontal stretches, and other distortion mappings

NUMERICAL METHODS (N)

4. RATIONAL NUMBERS

a) Operations with fractions

b) Reciprocals as multiplicative inverses

c) Converting fractions to decimals, terminating or repeating

d) Number patterns when rational numbers are in decimal form; use of calculators in forming patterns

e) Rounding errors arising in the use of calculators

f) Irrational numbers as non-terminating, non-repeating decimals; real numbers

5. †† APPLICATIONS OF PERCENT, RATIO, MEASUREMENT

a) Problems from business and commerce involving percent

b) Reductions and enlargements as percentage changes

c) Measurement and interpretation of scale drawings and models

d) Solution of real world problems using ratio and rate

e) Numerical investigation of compounding situations such as growth, decay, interest

f) Measurement of surface area of solids, including pyramids and cones; surface area related to nets

g) Measurement of volume of solids, including pyramids and cones; volume relationship between a cone and cylinder, between a pyramid and prism

ALGEBRA (A)

4. RELATIONS

a) Representing relations by arrow diagrams; mappings; ordered pairs

b) Graphing relations from the real world

c) The linear relation and its graph

d) Solving two linear relations graphically; algebraically

5. PROBABILITY

a) Calculating probability using tree diagrams

b) Probability related to games of chance

c) Counting problems in real life situations

d) Definition of probability

GEOMETRY (G)

3. †† CONGRUENCE TRANSFORMATIONS

a) Reflection, translation, and rotation images constructed from their fundamental properties; properties of these transformations

b) Given two congruent figures: tests for the type of transformation; construction of the reflection-line, translation arrow, or rotation centre and angle

c) Half-turn; properties; point-symmetry; applications

d) Glide reflection; properties

e) Isometries; meaning of congruence; direct and opposite congruence; finding the isometry for two congruent figures

4. DILATATION

a) Constructing dilatation images with scale factor k when $k > 1$, $0 < k < 1$, and $k < 0$

b) Properties of dilatation; ratios of corresponding sides, and of corresponding areas; similar figures

c) Investigating numerically the slope and length of a segment under a dilatation, using coordinates

d) Tiling patterns to illustrate the properties of dilatation

e) Testing to show that two figures are related by a dilatation

NUMERICAL METHODS (N)

6. MATRICES

a) Systematic display of numerical data by lists and tables; matrices

b) Use of row and column notation to locate elements

c) Dimensions of a matrix; addition of matrices; applications

ALGEBRA (A)

GEOMETRY (G)

5. DISPLACEMENT

a) Representing a displacement by an arrow, a trip to successive places by a succession of arrows; resultant displacement

b) Representing a displacement by an ordered pair, a trip to successive places by more than one ordered pair; resultant displacement

c) Representing trips by distance and bearings

d) Investigating travel routes on a globe; intuitive notions of the need for spherical geometry for world and space travel

NOTE: †† indicates sections which are to be fully implemented as printed beginning in September 1981.

LEVEL 5 GRADE 10 MATHEMATICS (ADVANCED)

OUTLINE OF TOPICS

NOTE 1. *This is a DRAFT COPY for validation purposes and for optional implementation during the school years 1977 – 1978 and 1978 – 1979. Reactions should be forwarded to the Intermediate Mathematics Coordinator, Ministry of Education, Curriculum Branch, Mowat Block, Queen's Park, Toronto M7A 1L2.*

NOTE 2. *In designing courses at the local level, topics within the dashed rectangles [ ] are to be considered optional; all other topics are core. Sections coded by ††† are to be fully implemented as printed beginning in September 1982.*

September 1977

NUMERICAL METHODS (N)

1. RADICALS

a) Entire and mixed radicals

b) Radical factors

c) Computation with radicals

2. APPLIED TRIGONOMETRY

a) Activities that relate ratios of sides with angles of right angled triangles

b) Sine, cosine, and tangent of an angle

c) Solving problems involving right angled triangles; angles of elevation and depression

3.†††STATISTICS

a) Experiments that lead to the concept of data forming a distribution

b) Experiments in which statistical measures of samples are compared to the corresponding measures of the population

c) Simple inferences

d) Predictions based on sample polls; reliability of predictions

e) Histogram; distinction between bar graph and histogram; class boundaries for histograms

f) Cumulative frequency tables and curves

g) Deciles; percentiles

ALGEBRA (A)

1. EQUATIONS AND INEQUATIONS

a) Graphing a linear equation, inequation

b) Graphing two linear equations, two or more inequations; intersection set

c) Solving a system of two linear equations algebraically

d) Solving systems in c) using matrices

e) Applications of preceding topics in practical situations

f) Linear programming

g) Graphing linear equations expressed in parametric form

2. RELATIONS

a) Given various graphs, identifying which ones represent functions

b) Graphing the circle, ellipse, and parabola from tables of values

c) Recognition of the above figures in architectural designs and in other art forms

GEOMETRY (G)

1. COORDINATE GEOMETRY

a) Slope of a segment; applications

b) Slopes of parallel segments, perpendicular segments

c) Distance between points; applications

d) Midpoint of a segment determined in numerical cases

e) Properties of triangles and quadrilaterals, illustrated using numerical coordinates

f) Investigating real world situations related to direct variation, and partial variation; graphical representation of the data

g) Graphing equations of the form $y = x$, $y = mx$, $y = x + b$, $y = mx + b$; applications

h) Fitting a linear function to the data in f)

2.†††COORDINATES AND TRANSFORMATIONS

a) Developing mapping rules for the image of a point (x, y) under translation, special cases of reflection and rotation, and dilatation

b) Investigating a segment and its image under the mappings in a)

c) Determining the equation of the image of a line under the mappings in a)

d) Using a 2 x 2 matrix to find images of points under certain mappings

NUMERICAL METHODS (N)

4. MATRICES

a) Storing information in matrices

b) Addition of matrices

c) Multiplication of matrices

d) The transpose matrix

e) The identity matrix for multiplication

f) The inverse of a 2 x 2 matrix

g) Use of the inverse matrix in solving pairs of simultaneous equations

h) The matrix as an operator in transformation geometry

i) The matrix as a representation of a vector

j) Addition of vectors in their matrix form

ALGEBRA (A)

3. PROBABILITY

a) Concept of a sample space developed from tree diagrams

b) Probability of occurrence and of non-occurrence

c) Dependent and independent events

4. COMPUTERS

a) History and social implications of computers; possible futuristic developments

b) Decision boxes and branching

c) Iterative techniques

d) Study of a computer language

e) Application of flow charting in computer programming, mentally or diagrammatically

f) Using flow charts in solving problems and generating tables

g) Testing programs by numerical methods

5. POLYNOMIALS

a) Multiplication with polynomials

b) Factoring trinomials and difference of squares

c) Operations with rational expressions

GEOMETRY (G)

3.††† ISOMETRIES

a) Glide reflection; properties

b) Isometries; invariant properties; direct and opposite congruence; determining the isometry for two congruent figures

c) Successive reflections in two parallel lines as a translation, and in two intersecting lines as a rotation; fundamental properties of each

d) Composite of three reflections related to glide reflection

e) Investigating other cases of the composition of two or more isometries

4.††† DEDUCTIVE GEOMETRY

a) Developing local axiomatic systems related to congruent figures, parallel lines, and angles of triangles and polygons; using techniques of traditional Euclidean Geometry, or isometries, or a combination of both

5. VECTORS

a) Vector quantity; vectors represented by arrows (directed line segments); equivalent arrows for one vector; vectors represented by ordered pairs

b) Addition of vectors by the triangle law

c) Multiplication by a scalar

d) Applications to problems related to navigation and force

e) Geometric properties derived by vector methods

f) Vectors in three-dimensions represented by arrows, and by ordered triples; vector addition by successive applications of the triangle law using arrows and/or triples; applications to force

NUMERICAL METHODS (N)

ALGEBRA (A)

GEOMETRY (G)

6. THREE-DIMENSIONAL GEOMETRY

a) Construction of cylinders and cones from their nets; surface area and volume of cylinders, cones, and spheres

b) Applications of a) to packaging of solids and liquids

c) Examples from the real world of plane symmetry, rotational symmetry, and point symmetry

d) Examples from the real world of 3-D to 2-D mappings; projections

e) Other topics from Grade 7 Mathematics, Geometry Section 6 and Grade 8 Mathematics, Geometry Section 5

NOTE: ††† indicates sections which are to be fully implemented as printed beginning in September 1982.

THE PROGRAM FOR LEVEL 4 (GENERAL)

The Level 4 program is designed for students who

- have completed Grade 8 Mathematics, but have experienced difficulty in consolidating and applying the skills and concepts of the Grades 7 and 8 program, and in understanding abstractions
- are planning to study *Applications of Mathematics* courses in the Senior Division
- do not intend to study Grade 13 mathematics, or to study mathematics or subjects in which mathematics is required at the university level
- may be planning to attend a College of Applied Arts and Technology or other non-university educational institution and to take courses requiring mathematics

The school's program is to consist of courses that are developed locally from the Outlines of Topics for Level 4, as provided on pages 36-41, and are consistent with the policy and directions established by this guideline. The general ground rules for this development are discussed in the section entitled Core and Options, found on page 9. These courses should recognize that students who select this program, generally speaking, will have experienced difficulty with their earlier studies in mathematics. They may be uncertain of their knowledge of basic skills and concepts and of their ability to apply this knowledge. They probably are uncomfortable when trying to make generalizations from specific situations, and have a limited understanding of mathematical abstractions. For these reasons they may lack motivation and be discouraged by their previous lack of success.

Different students will have different strengths and weaknesses, and so it is important that teachers provide sufficient flexibility in their courses to accommodate their varying needs. The courses should make liberal use of "hands-on" activities and of practical applications that are related to the students' personal experiences. Ample time should be allowed for establishing essential skills and concepts. Extensions of these ideas in breadth, depth, varieties of approaches, and sophistication of applications and abstractions should be treated cautiously, if at all, and should be attempted only with those students who appear ready. Above all, it is important that the program be interesting to the students and provide them with opportunities for successful completion of the work. In this way, self-confidence will be enhanced and students will approach mathematics with greater enthusiasm.

The chart below summarizes the major topics on which courses for Level 4 are to be developed.

LEVEL 4 (GENERAL)

	GRADE 9 MATHEMATICS (GENERAL)			GRADE 10 MATHEMATICS (GENERAL)		
SECTION	NUMERICAL METHODS	ALGEBRA	GEOMETRY	NUMERICAL METHODS	ALGEBRA	GEOMETRY
1.	Number Applications	*Flow Charts ††	*The Circle	Number Applications	*Relations	*Geometric Figures and Properties
2.	*Statistics ††	*Equations	*Coordinate Geometry	Ratio and Rate	*Linear Equations and Inequations	Coordinate Geometry
3.	*Powers, Roots	Polynomials	Congruence Transformations	†††Statistics	Polynomials	Vectors
4.	*Rational Numbers	Relations	*Dilatation	*Square Roots		
5.	Percent	Probability	Displacement	Personal Finance		
6.	*Applications of Ratio and Measurement			Payrolls and Flow Charts		
7.	Lists, Tables			Applied Trigonometry		
8.				Matrices		

NOTE:

i) Optional Sections — [ ] *indicates sections in which all the topics are optional*

ii) Optional Topics — * *indicates sections in which some of the topics are optional.*

iii) Delayed Implementation — †† *indicates sections that are to be fully implemented as printed beginning in September 1981.*
††† *indicates sections that are to be fully implemented as printed beginning in September 1982.*

A comparison of the Level 4 Grade 9 course with the Level 5 Grade 9 course found on page 24 shows that the section headings are almost the same. A study of the detailed Outlines of Topics for the two courses, however, shows that the students in the Level 5 program are expected to study a more comprehensive core program. In addition, the scope of the classroom development of the Level 5 course is intended to be more extensive – greater breadth and depth, including generalizations and abstractions; more variety in the approaches and methodologies that are used; more sophisticated examples and applications; and higher expectations for student performance. The Level 5 course should be considered essentially as an enriched Level 4 course, with both courses featuring concrete activities and an emphasis on relevant real-life applications. In the Level 4 course, students will need more time at the experiential level, time to assimilate the concepts and to practice the skills. It is not necessary for them to extend these experiences to the abstraction stage.

As a real-life application of percent, the topic *interest* is used to illustrate the strategy suggested above. Initial experiences may deal with bank accounts, Christmas clubs, lay-away plans, savings bonds, and so on. Students could check the interest payments for their own personal financial activities; however, contrived situations using realistic materials such as bank deposit forms are equally effective. Then combinations of deposits, withdrawals, changes in interest periods, and rates could be considered. At this stage, the topic of interest on the interest might be introduced as repeated multiplication. Students in the Level 4 program would consider some of the above applications, while students in Level 5 could be expected to generalize these ideas into the formula $A = P(1 + i)n$, and then to look at other situations which are of a similar mathematical nature, such as population growth and carbon dating.

This organizational strategy of these Grade 9 courses is designed to facilitate the transfer of students between Levels 4 and 5, when appropriate, with a minimum of disruption in each student's program. Also, it makes it easier for teachers to adjust the programs of individual students during the year, when a mid-year transfer is impossible due to conflicts in the timetable.

A similar organization for the Level 4 and Level 5 Grade 9 courses does not apply in Grade 10, by which time it is assumed that most students will have settled on the appropriate level for their personal needs. In Grade 10, the Level 4 program develops a stronger applications orientation and the core content is reduced to an estimated 60 percent of the year's work. The selection of options makes it possible to slant the course toward students with an interest in business subjects, or technical subjects. Options other than those listed may also be used at the discretion of the teacher.

Statistics and Flow Charts have been designated for delayed implementation as printed until 1981 in Grade 9 and 1982 in Grade 10. Their development depends on a sequential build-up through Grades 7 and 8. In the interim period, modified sections are expected to be developed using some of the ideas from earlier grades.

The full impact of the complete development of the new K – 8 program will not be known until some time in the 1980's. Teachers of Grades 9 and 10 in Level 4 should plan on their programs evolving at least for the next five years, as the incoming students gradually acquire a more complete foundation in the K – 8 program.

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LEVEL 4 GRADE 9 MATHEMATICS (GENERAL)

OUTLINE OF TOPICS

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NOTE 2. *In designing courses at the local level, topics within the dashed rectangles [ ] are to be considered optional; all other topics are core. Sections coded by †† are to be fully implemented as printed beginning in September 1981.*

September 1977

NUMERICAL METHODS (N)

1. NUMBER APPLICATIONS

a) Practising computational skills with whole numbers, decimals, and integers using games, puzzles, and activities; applications in real life situations

2. †† STATISTICS

a) Data gathering methods

b) Organizing data into frequency tables

c) Representing data in graphical form

d) Sample types

e) Practical experience in data surveying

f) Determining the mean, median, and mode

g) Appropriateness of mean, median, and mode as measures of central tendency

3. POWERS, ROOTS

a) Decimal approximations of square roots; applications in real life situations

b) Newton's method for calculating square roots

c) Using the Pythagorean theorem to solve problems in practical situations

d) Powers with integral exponents, their evaluation

e) Laws for multiplication and division with powers developed from numerical cases

f) Scientific notation; applications in other disciplines

ALGEBRA (A)

1. †† FLOW CHARTS

a) Flow charting as an organizational procedure

b) Loops for repetitive sequences of operations; application to solving problems and generating sequences

c) Efficient algorithms for use with calculators

2. EQUATIONS

a) Solving linear equations in one variable by systematic trial, and by the formal method

b) Manipulating formulae; applications

c) Solving word problems

d) Developing formulae from diagrams, numerical problems, and measurement activities

3. POLYNOMIALS

a) Adding and subtracting polynomials

b) Product of a monomial and a polynomial

c) Common factors

GEOMETRY (G)

1. THE CIRCLE

a) Basic constructions and their applications

b) Lines and circles related to triangles

c) Investigating properties related to a circle

d) Solving numerical problems using properties in b) and c)

2. COORDINATE GEOMETRY

a) Drawing figures on a grid given the coordinates of their vertices

b) Images of the figures in a) under reflection, translation, rotation, dilatation

c) Given a figure on a squared grid, drawing its images on a distorted grid

d) Experimenting with inclined planes the notion of slope; applications in the real world

e) Length of a line segment using the Pythagorean Theorem; applications in the real world

3. CONGRUENCE TRANSFORMATIONS

a) Constructing reflection translation, and rotation images; properties; symmetry

b) Investigating half-turn; properties; applications

NUMERICAL METHODS (N)

4. RATIONAL NUMBERS

a) Operations with fractions; reciprocals

b) Rational numbers in decimal form

c) Number patterns when rational numbers are in decimal form, use of calculators in forming patterns

5. PERCENT

a) Consolidation and extension of the concept of percent; fraction, decimal, and percent conversions; practice

b) Fractional percents; percents greater than 100 percent

c) Applications to real world situations

6. APPLICATIONS OF RATIO AND MEASUREMENT

a) Using ratio and rate concepts to solve real world problems

b) Measurement and interpretation of scale drawings, maps, and models

c) Investigating situations such as growth, decay, interest

d) Measurement of surface area of solids, including pyramids and cones; surface area related to nets

e) Measurement of volume extended to include pyramids and cones; volumes relationship between a cone and cylinder, between a pyramid and prism

7. LISTS, TABLES

a) Display of numerical data by lists, tables

b) Locating elements by row and column

c) Combining arrays with the same dimensions

ALGEBRA (A)

4. RELATIONS

a) Representing relations by diagrams, tables, and ordered pairs

b) Graphing relations

c) The linear relation and its graph

5. PROBABILITY

a) Calculating probability using tree diagrams

b) Probability related to games of chance

c) Counting problems from real life situations, and associated probabilities

GEOMETRY (G)

4. DILATATION

a) Constructing images on grids with scale factor k when $k > 1, 0 < k < 1$

b) Ratios of corresponding sides, corresponding areas; other properties; similar figures

c) Applications of enlargements and reductions in the real world; scale drawings

d) Recognizing similar figures and their properties in tiling patterns

5. DISPLACEMENT

a) Representing a displacement by an arrow, a trip to successive places by a succession of arrows; resultant displacement

b) Repeating a) using ordered pairs

c) Representing a trip by distance and bearing

NOTE: †† indicates sections that are to be fully implemented as printed beginning in September 1981.

LEVEL 4 GRADE 10 MATHEMATICS (GENERAL)

OUTLINE OF TOPICS

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NOTE 2. *In designing courses at the local level, topics within the dashed rectangles [ ] are to be considered optional; all other topics are core. Sections coded by ††† are to be fully implemented as printed beginning in September 1982.*

September 1977

LEVEL 4 GRADE 10 MATHEMATICS (GENERAL)

NUMERICAL METHODS (N)

1. NUMBER APPLICATIONS

a) Consolidation of number skills using games, puzzles, and activities; applications in real life situations

b) Rounding of whole numbers and decimals; errors resulting from successive calculations, in particular with calculators; applications in real life situations

c) Approximating and estimating

d) Interpolating and extrapolating tabulated data

e) Powers with integral exponents; scientific notation; applications in other disciplines

2. RATIO AND RATE

a) Ratios with two or more terms; practical examples

b) Proportions with two term ratios, and three term ratios; practical examples

c) Rate, practical examples

d) Direct variation; graphs and practical examples

e) Inverse variation; graphs and practical examples

3.†††STATISTICS

a) Histogram; distinction between bar graph and histogram; frequency polygon and frequency curve

b) Class width and class boundaries

c) Cumulative frequency tables, polygons and curves

d) Deciles, quartiles, and percentiles

e) The frequency curve as a measure of likelihood

ALGEBRA (A)

1. RELATIONS

a) Numerical relations as ordered pairs

b) Graphing ordered pairs from realistic situations and from experiments; interpolating and extrapolating from the graphs

c) Graphing non-linear relations defined by equations

d) Functions; the vertical line test

2. LINEAR EQUATIONS AND INEQUATIONS

a) Experiences and activities that help form the concepts of slope and intercept

b) The roles of m and b in equations of the form $y = mx + b$

c) Solution of real world problems that can be represented by linear equations

d) Graphing pairs of linear equations in two variables to determine the intersection; the intersection as the solution

e) Algebraic solution of two linear equations

f) Graphing linear inequations; boundaries and region of solution

g) Linear programming

3. POLYNOMIALS

a) Adding and subtracting polynomials

b) Product of a monomial and a polynomial

c) Common factors

d) Square of a binomial

GEOMETRY (G)

1. GEOMETRIC FIGURES AND PROPERTIES

a) Circle constructions, patterns and properties; applications in the real world

b) Properties related to parallel lines, perpendicular lines, triangles and quadrilaterals, and the Pythagorean Theorem; numerical exercises and applications

2. COORDINATE GEOMETRY

a) Experimenting with inclined planes; the notion of slope; applications in the real world

b) Slopes of parallel segments, and perpendicular segments

c) Length of a line segment using the Pythagorean Theorem; applications in the real world

3. VECTORS

a) Vectors represented by arrows (directed line segments); equivalent arrows for one vector; vectors represented by ordered pairs

b) Addition of vectors by the Triangle Law

c) Multiplication by a scalar

d) Applications to problems related to navigation and physics; relative motion

NUMERICAL METHODS (N) | ALGEBRA (A) | GEOMETRY (G)

4. SQUARE ROOTS
   a) Estimating square roots
   b) Newton's method for calculating square roots
   c) Applications of the Pythagorean Theorem
   d) Formulae involving square roots; applications

5. PERSONAL FINANCE
   a) Cost of credit
   b) Instalment purchases; leases; deferred payment schemes
   c) Savings and lending institutions
   d) Automobile ownership

6. PAYROLLS AND FLOW CHARTS
   a) Organizing calculations using flow charts
   b) Generating payroll summaries using flow charts
   c) Checking personal income records

7. APPLIED TRIGONOMETRY
   a) Activities that relate ratios of sides with angles of right angled triangles
   b) Sine, cosine, and tangent of an angle
   c) Solving problems involving right angled triangles; angles of elevation and depression

8. MATRICES
   a) Storing numerical data in matrix form
   b) Addition of matrices representing real world situations
   c) Multiplication of matrices representing real world situations
   d) Using matrices in Game Theory

NOTE: ††† indicates that this section is to be fully implemented as printed beginning in September 1982.

THE PROGRAM FOR LEVEL 3 (BASIC)

The Level 3 program is designed for students who

- have been promoted from elementary school based on their total achievement, but have not completed the regular mathematics program for Grades 7 and 8
- require additional experiences with the concepts, skills, and applications of Grades 7 and 8 Mathematics before attempting other mathematics courses
- may not be planning to study mathematics beyond the two credits in mathematics required during the first two years of the secondary school

This program is to consist of courses that are developed locally from the Outlines of Topics for Level 3, as provided on pages 44-48, and are consistent with the policy and directions established by this guideline.

Most of the students who enrol in the Level 3 Grade 9 course will have experienced much frustration in their attempts to learn mathematics, to the point that many of them will feel defeated even before beginning this course. There will be a wide spectrum of ability beneath a communal lack of motivation to learn mathematics, and many of the students will have anxieties in facing the prospect of another course in mathematics – this time in the new and strange environment of a secondary school.

These conditions suggest that an approach be used that will maximize the attention given to individual differences, provide opportunities for success, and minimize the anxieties held by many of the students. This strategy should be student-centred and experiential. It should evolve from a generalized problem-solving model that provides a focus for classroom activities and stresses the testing of solutions against objectives and reasonableness of answers.

The mathematics should be presented in simple form and in a mode befitting the social maturity of the students. It should be based on the students' experiences in the real world and make use of materials with which the students personally relate. These practical experiences should tie the essential skills and concepts of the program to applications that are real to the students now or can be accepted as of value in the future. The chart below summarizes the major topics on which the Level 3 courses are to be developed.

LEVEL 3 (BASIC)

GRADE 9 MATHEMATICS

SECTION	NUMERICAL METHODS	ALGEBRA	GEOMETRY
1.	*The Counting Numbers	*Problem Solving	Basic Notions Of Geometry
2.	Computation with Whole Numbers and Decimals	Introduction to Computing	Constructions
3.	Fractions	The Mathematics of Chance	Transformations
4.	Ratio and Rate		3-D Geometry
5.	Percent		
6.	Data Presentation		
7.	Powers and Roots		
8.	Integers		
9.	*Measurement		

NOTE:

i) Optional Sections [ ] indicates sections in which all the topics are optional.

ii) Optional Topics * indicates sections in which some of the topics are optional.

GRADE 10 MATHEMATICS

This course is to be based on the topics of Level 3, Grade 9 Mathematics (Basic) developed in the context of relevant applications of the topics

It is suggested that these applications be developed from Themes, such as those outlined in:

i) *Notes for Teachers, Level 3, Grade 10 Mathematics (Basic)*

ii) *Appendix 1, Level 3, Grade 9 Mathematics (Basic)* – Themes

iii) *Appendix 2, Level 3, Grade 9 Mathematics (Basic)* – a sample development of thematic materials

iv) Other resource materials related to Level 3

Other themes may be developed locally.

Themes and/or subthemes used in the Grade 10 course should not duplicate these used to develop the Grade 9 course.

The Grade 9 course is intended for two groups of students – those who will enrol in the Level 4 Grade 9 course in the following year, and those who will take the Level 3 Grade 10 course. Both groups are weak in the skills and concepts of elementary mathematics and need additional time and experiences to consolidate these before going on to further studies.

All the major core topics, and many of the options, of the Grade 9 course can be found in the elementary school program. It is expected, however, that these topics will be developed in new contexts and at a relaxed pace, so that the students of the Level 3 program will feel they are taking new work.

The Notes For Teachers for Grade 9 are intended to help in the design of courses meeting the above criteria. They expand the Outlines of Topics by giving detailed suggestions of appropriate levels of development for each topic and by listing other topics, concepts, and skills that are related to each topic in the course.

Appendices 1 and 2 will help teachers to design thematic approaches to this course. Appendix 1 identifies 15 themes and many subthemes within them, through which the concepts and skills of the course can be identified and developed. From this resource, courses may be planned that feature a number of thematic studies. Classroom activities and materials will need to be developed for this purpose. Appendix 2 is a sample development of a subtheme, ready for use by students. During the classroom development of these themes, a number of applications of mathematical concepts and skills should emerge. When necessary, mini-lessons should be taught to those students who need help with these concepts and skills. Review materials and practice exercises should be used to consolidate the fundamental ideas that are needed by these students.

A program that is organized in this way will feature real world experiences from which mathematical ideas will emerge and from which the need for learning and consolidating basic mathematical ideas will follow. The same skills and concepts will emerge in a number of the subthemes; in this way students will have many occasions in entirely different contexts to recall, practice, consolidate, and apply the same fundamental ideas. This development can be intrinsically interesting to the students and motivational levels may rise and anxiety levels fall.

The program for these students must provide success if the students' interest and enthusiasm is to be maintained. Evaluation will need to be frequent, varied, both objective and subjective, and above all encouraging.

The core topics in the Grade 9 course are provided as a suggestion only, and are not binding. They are the topics that the students should know to proceed with further studies. Because of the nature of the students and their highly individual needs, the expectation that all students must study the same topics to the same level of proficiency would be self-defeating and would destroy the humane features this course is intended to establish. Even though the course at the planning stage should include all of the core topics, at the classroom level it should be interpreted with flexibility that is responsive to the individual needs of students.

The Level 3 Grade 10 course is entirely applied. It builds upon the concepts and skills of the Level 3 Grade 9 course. The students are expected to study relevant applications of these topics and, in so doing, to consolidate their understanding of, and ability to apply, these basic mathematical ideas.

Again, this program can be developed by a thematic approach. The Notes For Teachers for Level 3 Grade 10 Mathematics provide 9 suggested themes and subthemes. These represent situations that the students have experienced, are re-experiencing, or will experience in adult life. They involve applications that use mathematical skills required by workers, consumers, citizens, and parents and other family members. Appendix 1 for this course provides a list of materials that are available from financial institutions in Ontario. These will be useful in developing Theme 3 of the Notes For Teachers.

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LEVEL 3 GRADE 9 MATHEMATICS (BASIC)

OUTLINE OF TOPICS

NOTE 1. *This is a DRAFT COPY for validation purposes and for optional implementation during the school years 1977 – 1978 and 1978 – 1979. Reactions should be forwarded to the Intermediate Mathematics Coordinator, Ministry of Education, Curriculum Branch, Mowat Block, Queen's Park, Toronto M7A 1L2.*

NOTE 2. *In designing courses at the local level, topics within the dashed rectangles ┆ ┆ are to be considered optional; all other topics are core.*

September 1977

NUMERICAL METHODS (N)

1. THE COUNTING NUMBERS

a) Counting; numerals and place value
b) Reading numerals, writing numerals in words
c) Understanding and using special numerical terms
d) The concepts of zero and one
e) Ordering and sequencing
f) The notion of a code; coding and decoding

2. COMPUTATION WITH WHOLE NUMBERS AND DECIMALS

a) Rounding
b) Addition, subtraction, multiplication, and division with whole numbers
c) Calculations related to money
d) Operations with decimals

3. FRACTIONS

a) The concept of fraction; mixed number
b) Equivalent fractions
c) Conversions and simplifications
d) Conversion of a fraction to a decimal
e) Multiplying and dividing with fractions and mixed numbers
f) Common denominator; addition and subtraction of fractions and mixed numbers

4. RATIO AND RATE

a) The concept of ratio; equivalent ratios
b) The concept of rate

ALGEBRA (A)

1. PROBLEM SOLVING

a) Analysing and solving problems
b) Aids to problem solving: understanding mathematical words, diagrams, charts and tables, formulae, flow diagrams, calculators
c) Equations: concept of variable, forming equations, solving equations

2. INTRODUCTION TO COMPUTING

a) Uses of a computer
b) Working with a computer
c) Binary System
d) Uses of flow charting

3. THE MATHEMATICS OF CHANCE

a) Probability in simple 2-outcome experiments
b) Probability in experiments with many possible outcomes
c) Sample space by listing
d) Sample space by a tree diagram
e) Size of sample space by The Fundamental Counting Principle
f) Sampling for composition
g) Probability based on experiments
h) Odds

GEOMETRY (G)

1. BASIC NOTIONS OF GEOMETRY

a) The language of geometry
b) Perpendicular and parallel lines
c) Symmetry
d) Properties of plane figures

2. CONSTRUCTIONS

a) Use of protractor
b) Accurate constructions using a variety of instruments

3. TRANSFORMATIONS

a) Slides, turns, and flips
b) Enlargements and reductions
c) Distortions

4. THREE-DIMENSIONAL GEOMETRY

a) Solids, shells, and skeletons
b) Polyhedra
c) Pyramids and prisms
d) Sketching 3-D objects

NUMERICAL METHODS (N)

5. PERCENT

a) The concept of percent

b) Conversion of percents to decimals, decimals to percents

c) Computation with percents

6. DATA PRESENTATION

a) Reading tables

b) Reading pictographs

c) Understanding bar, line, and circle graphs

d) Collecting data: attribute data, numerical data

e) Drawing graphs: pictographs, bar graphs, line graphs, circle graphs

7. POWERS AND ROOTS

a) The meaning of power notation

b) Square root as the inverse of square

8. INTEGERS

a) The concept of integer

b) The order of integers; the I-line

c) Operations with integers

9. MEASUREMENT

a) Metric units; suitability, conversion of units

b) Length; use of metric symbols mm, cm, m, km; estimating

c) Area; use of metric symbols mm^2, cm^2, m^2, km^2, ha; estimating

d) Volume; use of metric symbols mm^3, cm^3, m^3, km^3; estimating

e) Capacity; use of metric symbols mL, L; estimating

f) Mass; use of metric symbols mg, g, kg, t; estimating

ALGEBRA (A)

GEOMETRY (G)

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LEVEL 3 GRADE 10 MATHEMATICS (BASIC)

OUTLINE OF TOPICS AND THEMES

NOTE: *This is a DRAFT COPY for validation purposes and for optional implementation during the school years 1977 – 1978 and 1978 – 1979. Reactions should be forwarded to the Intermediate Mathematics Coordinator, Ministry of Education, Curriculum Branch, Mowat Block, Queen's Park, Toronto M7A 1L2.*

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LEVEL 3 GRADE 10 MATHEMATICS (BASIC)

1. **OUTLINE OF TOPICS**

This course is intended to further reinforce and develop the topics introduced in the *Outline of Topics* for *Level 3, Grade 9 Mathematics (Basic).* However, the core-option designation of topics in the Grade 9 course does not apply here. The selection of topics should depend on local conditions and the specific needs of the students and, at the teachers' discretion, may be extended beyond the topics of the Grade 9 course.

It is intended that the course stress applications of mathematics that are relevant to the students now and will be useful to them in the future. To this end, it is recommended that the program be developed from themes which involve applications of the skills and concepts introduced in earlier courses.

2. **OUTLINE OF THEMES**

The following themes* are given as a starting point for a thematic development of this course. This list may be extended by themes from *Appendix 1, Level 3, Grade 9 Mathematics (Basic)* and by other locally developed themes.

Theme 1.	Math at Work and at Home
Theme 2.	Your Earnings and Your Budget
Theme 3.	You and Financial Services
Theme 4.	You and Taxes
Theme 5.	You and Travelling — Public and Private
Theme 6.	You and Insurance
Theme 7.	Mathematics in Sports
Theme 8.	Setting Up A Place of One's Own
Theme 9.	You and The Music World

* See the *Notes for Teachers, Level 3, Grade 10 Mathematics (Basic); Appendices 1 and 2* for *Level 3, Grade 9 Mathematics (Basic);* and *Appendix 1* for *Level 3, Grade 10 Mathematics (Basic).*

THE PROGRAM FOR LEVELS 1 AND 2

The Level 2 program is designed for students who

- are taking special vocational or occupations courses
- have had exposure to at least Grade 7 Mathematics
- have understanding of Junior Division Mathematics

The Level 1 program is designed for students who

- are taking special vocational, occupations, or services courses
- have understanding of mathematics of the Primary or early Junior Divisions

The program is to consist of courses that are developed locally from Outlines of Topics A and B for Level 1, and B and C for Level 2, as provided on pages 51-63.

	LEVEL 2	LEVEL 1
GRADE 9	COURSE B	COURSE A
GRADE 10	COURSE C	COURSE B

The chart below summarizes the major topics on which Courses A, B, and C are to be developed.

LEVELS 1, 2

SECTION	COURSE A LEVEL 1 GRADE 9 MATHEMATICS NUMERICAL METHODS	GEOMETRY	COURSE B LEVEL 2 GRADE 9 MATHEMATICS LEVEL 1 GRADE 10 MATHEMATICS NUMERICAL METHODS	GEOMETRY	COURSE C LEVEL 2 GRADE 10 MATHEMATICS LEVEL 1 GRADE 11 MATHEMATICS NUMERICAL METHODS	GEOMETRY
1.	*Non-Computational Use of Numbers	Motion Geometry	Non-Computational Use of Numbers	Motion Geometry	Non-Computational Use of Numbers	*Motion Geometry
2.	*Computational Use of Whole Numbers	*Geometric Constructions	Computational Use of Whole Numbers	*Geometric Constructions	Computational Use of Whole Numbers	*Geometric Constructions
3.	Computational Use of Decimals	Prisms and Pyramids	Computational Use of Decimals	Prisms and Pyramids	Computational Use of Decimals	Prisms and Pyramids
4.	Percent		*Percent		*Percent	
5.	Aids to Solving Mathematical Problems		Aids to Solving Mathematical Problems		Problem Solving	
6.	*Fractions		Fractions		Fractions	
7.	*Data Graphs		*Data Graphs		*Data Graphs	
8.	*Measurement		*Measurement		*Measurement	
9.	Probability		*Probability		*Probability	

NOTE:

i) Optional Sections [ ] *indicates sections in which all the topics are optional.*

ii) Optional Topics * *indicates sections in which some of the topics are optional.*

These courses are to be consistent with the policy and directions established by this guideline.

Students in these two programs usually have difficulty in learning and maintaining the basic skills and concepts that are needed for everyday living, now and in the future. As a result, essentially the same topics are offered in Courses A, B, and C – but the depth and breadth of treatment and the contexts in which these skills are to be identified and applied are expected to change from Course A to Course B to Course C. In this way the essential skills and concepts are presented from a different viewpoint each year. This fresh approach will help students to maintain interest in the program.

The core topics in each of these courses are presented as ideals to be worked toward. They should be considered during the planning of courses. However, it is unrealistic to expect that all students will be ready to study all topics of the core program, because of the variations of their strengths and weaknesses. Comprehensive accounts of the work attempted by each student as well as a description of the contexts in which it was presented are necessary for teachers of subsequent courses. These will assist them in planning courses that build on earlier experiences and enable students to progress according to their individual abilities.

The Notes For Teachers for courses A, B, and C suggest some contexts that may be used for developing the topics. Teachers could use these as starting points for extending the lists of possible contexts. The Notes For Teachers also identify other topics, concepts, and skills that are related to the topics under discussion. These connections may be used at appropriate points in the development of each topic; they provide opportunities to recall and consolidate past experiences and to introduce new ideas as the course progresses.

It is important that each course stress applications that are relevant to the students. This will require the collection and organization of special materials by individuals or cooperatively by groups of teachers. Students need to be able to identify with the materials used, so that they can recognize utilitarian values in the day-to-day school activities.

The classroom environment can be made non-threatening by the setting of realistic expectations so that students will be encouraged by the likelihood of success. The school has the major responsibility for ensuring that students learn to use mathematics in situations that will be important to them throughout their lives.

An effective approach for this program involves the development of themes that are within the students' experiences. This approach generates interest and motivation for the students and shows them how mathematics is used on a day-to-day basis in the real world. Themes provide a natural vehicle for involving students in problem-solving techniques, and for identifying those students who need extra help with their basic skills and/or conceptual understandings.

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COURSE A

LEVEL 1 GRADE 9 MATHEMATICS

OUTLINE OF TOPICS

NOTE 1. *This is a DRAFT COPY for validation purposes and for optional implementation during the school years 1977 – 1978 and 1978 – 1979. Reactions should be forwarded to the Intermediate Mathematics Coordinator, Ministry of Education, Curriculum Branch, Mowat Block, Queen's Park, Toronto M7A 1L2.*

NOTE 2. *In designing courses at the local level, topics within the dashed rectangles [ ] are to be considered optional.*

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NUMERICAL METHODS (N)

1. NON-COMPUTATIONAL USE OF NUMBERS

a) Counting from 1 to 100 by 1, 2, 5, 10, 20, 25

b) Writing numbers from 1 to 100 using numerals, words

c) Reading numerals to 1 000 000

d) Arranging numbers in order; smallest to largest; largest to smallest

e) Ranking things as 1st, 2nd, 3rd, etc.; smallest to largest, largest to smallest

f) Coding: simple codes; identifying parts of a code; coding and decoding

2. COMPUTATIONAL USE OF WHOLE NUMBERS

a) Place value: units, tens, hundreds, thousands

b) Adding pairs of numerals with up to 3 digits

c) Subtracting pairs of numerals with up to 2 digits

d) Multiplication facts; multiplying 1- and 2-digit numerals by 1-digit numerals

e) Factoring numerals; e.g., $6 = 2 \times 3$

f) Dividing by 2, 3, 4, 5, 6, and 10

g) Properties of addition and multiplication, developed informally

h) Reasonableness of answers

3. COMPUTATIONAL USE OF DECIMALS

a) Place value from hundreds to hundredths

b) Adding and subtracting pairs of decimals of the types in a)

c) Multiplying and dividing by 10, 100, 1000

d) Reasonableness of answers

e) Approximating

GEOMETRY (G)

1. MOTION GEOMETRY

a) Slides: drawing a slide image on a grid; identifying slide images on grids and in the real world

b) Flips: drawing a flip image on a grid; identifying flip images on grids and in the real world; symmetry

c) Turns: 1/4, 1/2, 3/4 and full turns; drawing a turn image on a grid; identifying turn images on grids and in the real world; symmetry

2. GEOMETRIC CONSTRUCTIONS

a) Angles: using a protractor to measure and to construct angles such as 90°, 45°, 60°, 30°; estimating the size of angles

b) Constructing circles using a compass; centre, radius, diameter

c) Constructing triangles using a variety of techniques; right angled, isosceles, equilateral, scalene

d) Regular polygons: using a compass and protractor to construct a square, equilateral triangle, and other regular polygons in a circle; symmetry

e) Tiling patterns; tiling with triangles, quadrilaterals, regular hexagons

3. PRISMS AND PYRAMIDS

a) Prisms: identified by the base; vertex, edge, face; description of prisms

b) Pyramids: identified by the base; vertex, edge, face; description of pyramids

NUMERICAL METHODS (N)

GEOMETRY (G)

4. PERCENT

a) Percent as hundredths; e.g. $45\% = \frac{45}{100} = 0.45$; application to finding percent of a number in real life situations

5. AIDS TO SOLVING MATHEMATICAL PROBLEMS

a) Using simple flow charts

b) Using formulae: mathematics as a language; common symbols; simple sentences; simple formulae

c) Using a calculator

6. FRACTIONS ϕ

a) Fraction as part of a whole, as part of a group

b) Adding and subtracting fractions with the same denominators

c) Multiplying a whole number by a fraction, a fraction by a fraction

d) Decimal equivalents of fractions such as 1/2, 1/5, 1/4 and 1/10

ϕ simple cases only, with denominators of 2, 4, 8, 3, 5, 10

7. DATA GRAPHS

a) Reading, comparing, and interpreting information from charts and graphs

b) Collecting and organizing data

c) Representing data by pictographs and bar graphs

NUMERICAL METHODS (N) GEOMETRY (G)

8. **MEASUREMENT** ϕ

a) Informal measuring of length, area, volume, capacity, and mass using non-standard units

b) Measuring length, estimating

c) Measuring area, estimating

d) Measuring volume, estimating

e) Measuring capacity, estimating

f) Measuring mass, estimating

g) Measuring temperature, estimating

h) Selecting the appropriate unit; equivalent units and conversion

ϕ using metric symbols cm, m, km, cm^2, m^2, ha, cm^3, m^3, mL, L, g, kg, $^{\circ}C$, and imperial units until they are no longer in common use

9. **PROBABILITY**

a) Counting the outcomes of experiments

b) Formula for simple probability

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COURSE B

LEVEL 2 GRADE 9 MATHEMATICS

and LEVEL 1 GRADE 10 MATHEMATICS

OUTLINE OF TOPICS

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NOTE 2. *In designing courses at the local level, topics within the dashed rectangles* ┌ ┐ *are to be considered optional.*

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NUMERICAL METHODS (N)

1. NON-COMPUTATIONAL USE OF NUMBERS

a) Counting from 1 to 1000 by 1, 2, 5, 10, 20, 25

b) Writing numbers from 1 to 1000 using numerals, words

c) Reading numerals

d) Arranging numbers in order; using numbers to sequence things

e) Ranking things as 1st, 2nd, 3rd, etc.

f) Coding: simple codes; identifying parts of a code; coding and decoding

2. COMPUTATIONAL USE OF WHOLE NUMBERS

a) Place value: units, tens, hundreds, thousands

b) Adding pairs of numerals with up to 4 digits

c) Subtracting pairs of numerals with up to 4 digits

d) Multiplying 1- to 3-digit numerals by 1- and 2-digit numerals; squaring numbers 1, 2, 3, . . ., 10

e) Factoring 1- and 2-digit numerals; perfect squares

f) Dividing by 1- and 2-digit numerals

g) Properties of addition and multiplication, developed informally

h) Reasonableness of answers

i) Approximating

j) Estimating

3. COMPUTATIONAL USE OF DECIMALS

a) Place value from thousands to thousandths

b) Adding and subtracting decimals of the type in a)

c) Multiplying pairs of decimals of the type in a)

GEOMETRY (G)

1. MOTION GEOMETRY

a) Slides: drawing a slide image on a grid, on plain paper; identifying slide images in diagrams and in the real world

b) Flips: drawing a flip image on a grid, on plain paper; identifying flip images in diagrams and in the real world; symmetry

c) Turns: 1/4, 1/2, 3/4, and full turns; drawing a turn image on a grid, on plain paper; identifying turn images in diagrams and in the real world; symmetry

d) Distortions: drawing a distortion image using a distorted grid

2. GEOMETRIC CONSTRUCTIONS

a) Angles: using a protractor to measure and construct angles from 0° to 90°; estimating the size of angles

b) Constructing circles using a compass; centre, radius, diameter

c) Constructing triangles using a variety of techniques; right angled, isosceles, equilateral, scalene

d) Constructing squares and rectangles using a variety of techniques

e) Regular polygons: using a compass and protractor to construct a square, equilateral triangle, and other regular polygons in a circle; symmetry

f) Tiling patterns; tiling with triangles, quadrilaterals, regular hexagons; symmetry

NUMERICAL METHODS (N)

3. COMPUTATIONAL USE OF DECIMALS (Cont.)

d) Dividing a decimal of the type in a) by a 1-digit numeral

e) Multiplying and dividing a decimal of the type in a) by 10, 100, and 1000

f) Reasonableness of answers

g) Approximating

h) Estimating

4. PERCENT

a) Meaning of percent

b) Applications of finding a percent of a number

c) Applications of finding the percent of a part to the whole

5. AIDS TO SOLVING MATHEMATICAL PROBLEMS

a) Using simple flow charts

b) Using formulae: mathematics as a language; common symbols; simple sentences; simple formulae

c) Using a calculator

6. FRACTIONS $\oint$

a) Fraction as part of a whole, as part of a group

b) Equivalent fractions, simple cases only

c) Adding and subtracting fractions with the same denominators

d) Multiplying a whole number by a fraction, a fraction by a fraction

e) Decimal equivalents of fractions such as 1/2, 2/5, 3/4, and 8/10

$\oint$ with denominators of 2, 4, 8, 3, 5, 10

GEOMETRY (G)

3. PRISMS AND PYRAMIDS

a) Prisms: identified by the base; vertex, edge, face; description of prisms; examples of prisms in the real world

b) Pyramids: identified by the base; vertex, edge, face; description of pyramids; examples of pyramids in the real world

NUMERICAL METHODS (N)

GEOMETRY (G)

7. DATA GRAPHS

a) Reading, comparing, and interpreting information from charts and graphs

b) Collecting and organizing data from real life situations, from experiments

c) Representing data by pictographs, bar graphs, and line graphs

d) Graphing on a coordinate grid

8. MEASUREMENT ϕ

a) Informal measuring of length, area, volume, capacity, and mass using non-standard units.

b) Measuring length, estimating

c) Measuring area, estimating

d) Measuring volume, estimating

e) Measuring capacity, estimating

f) Measuring mass, estimating

g) Measuring temperature, estimating

h) Selecting the appropriate unit; equivalent units and conversion

ϕ using metric symbols cm, m, km, cm^2, m^2, ha, cm^3, m^3, mL, L, g, kg, t, $^{\circ}C$, and imperial units until they are no longer in common use

9. PROBABILITY

a) Counting the outcomes of experiments

b) Formula for simple probability

c) Finding the probability using a tree diagram

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COURSE C

LEVEL 2 GRADE 10 MATHEMATICS

and LEVEL 1 GRADE 11 MATHEMATICS

OUTLINE OF TOPICS

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NOTE 2. *In designing courses at the local level, topics within the dashed rectangles [ ] are to be considered optional.*

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NUMERICAL METHODS (N)

1. NON-COMPUTATIONAL USE OF NUMBERS

a) Counting (unrestricted) by 2, 3, 4, 5, 10, 20, 25; notion of finite sequences, infinite sequences

b) Writing numbers (unrestricted) using numerals, words

c) Reading numerals

d) Arranging numbers in order; using numbers to sequence things

e) Ranking things as 1st, 2nd, 3rd, etc.

f) Coding: simple codes; identifying parts of a code; coding and decoding

2. COMPUTATIONAL USE OF WHOLE NUMBERS

a) Place value

b) Adding and subtracting numerals with up to 4 digits, in columns and in rows

c) Multiplying 1- to 4-digit numerals; squaring numbers 1, 2, 3, . . . , 10; cubing numbers 1, 2, 3, 4

d) Factoring 1- and 2-digit numerals; perfect squares and their square roots

e) Dividing by 1- to 3-digit numerals

f) Properties of addition and multiplication, developed informally

g) Reasonableness of answers

h) Approximating

i) Estimating

GEOMETRY (G)

1. MOTION GEOMETRY

a) Slides: drawing a slide image on a grid, on plain paper; identifying slide images in diagrams and in the real world

b) Flips: drawing a flip image on a grid, on plain paper; identifying flip images in diagrams and in the real world; symmetry

c) Turns: drawing a turn image on a grid, on plain paper; identifying turn images in diagrams and in the real world; symmetry

d) Enlargements, Reductions: drawing an enlarged or reduced image on a grid; size and shape; identifying enlarged or reduced images in diagrams and in the real world; scale drawings

e) Distortions: drawing a distortion image using a distorted grid

2. GEOMETRIC CONSTRUCTIONS

a) Angles: using a protractor to measure and to construct angles from 0° to 180°; estimating the size of angles

b) Constructing circles using a compass; centre, radius, diameter

c) Constructing triangles using a variety of techniques; right-angled, isosceles, equilateral, scalene

d) Constructing squares and rectangles using a variety of techniques

e) Regular polygons: using a compass and protractor to construct a square, equilateral triangle, and other regular polygons in a circle; symmetry

f) Tiling patterns: tiling with triangles, quadrilaterals, regular hexagons; geometric properties; mosaics; symmetry

NUMERICAL METHODS (N)

3. COMPUTATIONAL USE OF DECIMALS

a) Place value (unrestricted)

b) Adding and subtracting decimals in columns and in rows

c) Multiplying decimals

d) Dividing decimals by 1- to 3-digit numerals

e) Multiplying and dividing a decimal by 10, 100, 1000, etc. and by 0.1, 0.01, etc.

f) Reasonableness of answers

g) Approximating

h) Estimating

i) Rounding and truncating

4. PERCENT

a) Meaning of percent

b) Applications of finding a percent of a number

c) Applications of finding the percent of a part to the whole

5. PROBLEM SOLVING

a) Solving a variety of real world and mathematical problems, by a method such as:

- identifying the problem
- collecting and organizing the significant information
- analysing this information and developing a strategy
- executing the strategy and testing the reasonableness of the result
- presenting the solution if the result is reasonable, or repeating the above

GEOMETRY (G)

3. PRISMS AND PYRAMIDS

a) Prisms: identified by the base; vertex, edge, face; characteristics of prisms; examples of prisms in the real world; right prisms and symmetry

b) Pyramids: identified by the base; vertex, edge, face; characteristics of pyramids; examples of pyramids in the real world; right pyramids and symmetry

c) Constructing prisms and pyramids as shells, as skeletons

d) Stacking space

NUMERICAL METHODS (N)

GEOMETRY (G)

6. FRACTIONS ∮

a) Fraction as part of a whole, as part of a group

b) Equivalent fractions; reduction of fractions (simple cases only)

c) Adding and subtracting fractions with the same denominators, with different denominators

d) Multiplying a whole number by a fraction, a fraction by a fraction

e) Decimal equivalents of fractions, such as 1/2, 2/5, 3/4, and 1/3

∮ with denominators of 2, 4, 8, 3, 5, 10

7. DATA GRAPHS

a) Reading, comparing, and interpreting information from charts, graphs, and tables

b) Collecting and organizing data from real life situations, from experiments; polling, surveying, sampling

c) Representing data by pictographs, bar graphs, circle graphs, and line graphs

d) Representing data in tables; mean, median, and mode of the data

e) Graphing on a coordinate grid

NUMERICAL METHODS (N)

GEOMETRY (G)

8. **MEASUREMENT** $\oint$

a) Measuring length, estimating

b) Measuring area, estimating

c) Measuring volume, estimating

d) Measuring capacity, estimating

e) Measuring mass, estimating

f) Measuring temperature, estimating

g) Measuring pressure

h) Selecting the appropriate unit; equivalent units and conversion

$\oint$ using metric symbols cm, m, km, cm^2, m^2, ha, cm^3, m^3, mL, L, g, kg, t, $^\circ$C, kPa, and imperial units until they are no longer in common use

9. **PROBABILITY**

a) Counting the outcomes of experiments

b) Formula for simple probability

c) Finding the probability using a tree diagram

d) The Fundamental Counting Principle; applications

e) Odds

f) Dependent events; independent events

EVALUATION

PURPOSE

Evaluation of student achievement and program effectiveness must be an integral, ongoing part of the teaching-learning process. It can serve many functions:

- to provide feedback to the student regarding what has been mastered and what still needs to be learned
- to provide parents with information regarding student progress
- to verify the appropriateness of stated goals and objectives
- to provide information that will enable teachers to modify the program as needed
- to assess the quality and effectiveness of learning strategies and materials that have been used
- to encourage the student to further achievement

SOURCES OF INFORMATION

Because evaluation is concerned with the provision of information to facilitate all aspects of the teaching-learning process, it is necessary to use a wide range of sources of information. These might include:

- unit or topic pre-tests
- unit or topic post-tests
- quizzes related to specific objectives
- examinations at specific intervals
- suitable standardized tests
- projects
- systematic observations of attitudes and performance
- personal interviews
- records of previous achievements
- oral presentations to the class of solutions to problems and of mathematical applications

STRATEGIES

As effective learning is dependent on the learner's readiness and background, one should attempt, as a first priority, to establish a profile of individual attitudes and competencies. In the light of this information, student learning can be planned and paced according to a personally appropriate sequence of learning objectives. As learning progresses, there is a need to provide immediate and continuing feedback to students in order to help them monitor their own progress. In addition, a more general assessment of the degree to which broader outcomes have been attained throughout some significant part of the course is also an important component of evaluation.

Initial evaluation processes that are directed toward identifying characteristics of the learner depend on both static and dynamic sources of information. Individual records indicate performance levels at particular points in time in the learner's past development. These should be brought up to date through systematic observation of student teacher interaction and careful interpretation of test results. For this information to be most useful, a systematic record system is absolutely vital. Studies have shown that the practice of keeping "field notes" pays dividends. It helps teachers to focus on individuals and to look more closely at them. As a consequence, they become aware of aspects of attitude and behaviour that were formerly unnoticed. It helps students to develop a greater sense of acceptance of the fairness of evaluation and confirms their progress. It provides a sound context within which to interpret test and examination results and other data. This is vital in assessing student progress: for the student, his or her parents, and for the school.

Students seldom learn only one thing at a time. As a result, evaluation of achievement in mathematics has been handicapped by the misconception that the learning process is straightforward and sequential, and hence, easy to measure. This is not the case. Assessment practices must take into account the complexity of learning. They must deal with specific learning tasks, with degrees of understanding, with values and attitudes, as well as with facts, skills and concepts. Written tests or examinations, no matter how well they are constructed, do not provide all the information needed for a valid assessment. It is only after data from interviews, observations, projects, and tests have been sorted and studied that a sound professional judgement can be made.

Most of the teacher's assessments will be formative, aimed at providing frequent feedback to individuals or groups of students regarding their progress over the short-term. The teacher can then check whether or not learning is effective and provide for changes in pacing, in materials, in teaching methods, or in stated objectives.

At regular intervals there should be an accounting of the progress exhibited by students and an analysis of the effectiveness of all aspects of the program. It should be noted that the practice of using evaluation, examinations, and the threat of failure as factors in motivating students is sometimes counterproductive. In fact, the anxiety generated by such strategies often prevents students from doing their best and the results are not indicative of either the level of their achievement or of the quality of the program. By arousing the student's intrinsic interests in the subject and by providing a reasonable and systematic process of evaluation, one can create a learning environment in which each student can strive to achieve the full realization of his or her potential.

COURSE A

for

LEVEL 1 GRADE 9 MATHEMATICS

NOTES FOR TEACHERS

TOPICS

CONTEXTS IN WHICH THE TOPICS MAY BE DEVELOPED

RELATED TOPICS, CONCEPTS, SKILLS

MATERIALS AND RESOURCES

NOTE 1. *This is a DRAFT COPY for validation purposes and for optional implementation during the school years 1977 – 1978 and 1978 – 1979. Reactions should be forwarded to the Intermediate Mathematics Coordinator, Ministry of Education, Curriculum Branch, Mowat Block, Queen's Park, Toronto M7A 1L2.*

NOTE 2. *In designing courses at the local level, topics within the dashed rectangles [ ] are to be considered optional.*

September 1977

STRAND NUMERICAL METHODS

TOPICS	CONTEXTS IN WHICH TOPICS MAY BE DEVELOPED	RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
SECTION 1 NON-COMPUTATIONAL USE OF NUMBERS			
a) Counting from 1 to 100 by 1, 2, 5, 10, 20, 25	Counting should be handled using activities that are appropriate to the age and social maturity of the students. The following are examples of counting activities relevant to the students' experiences: • desks in the class, seats in the auditorium, cars in the parking lot, etc., number of boys, number of girls, number of friends; numbers on a calendar, days, weeks, months; hours, minutes, seconds; money; probability experiments; population counts; etc. • counting on a calculator • music (Count Basie, Lawrence Welk), rhythm, number of bars, bars of rest, etc.	• idea of larger, smaller • addition; subtraction (how much larger? smaller?) • idea of before, after, between • probability experiments	• calendar • clock • calculator • Hextat • Monopoly and other money games • probability games • other games • mechanical counters
b) Writing numbers from 1 to 100 using numerals, words	• writing cheques, legal documents • magic squares, games	• special names for numbers; e.g., pair, dozen, century, etc.	• newspaper • textbooks • magazines • magic squares
c) Reading numerals to 1 000 000	• numbers in the newspaper, magazines, on T.V., in textbooks from other subjects (geography, science, shops, etc.) • historical dates • population of cities, towns, etc. • catalogues	• place value	• newspaper • T.V. • textbooks • bingo • *Almanac* • *Guinness Book of Records*
d) Arranging numbers in order: smallest to largest, largest to smallest	• house numbers, locker numbers, calendar • reading rows and columns on a student timetable • sports data (newspaper, magazines, etc.); school sports; sports records; scoring sporting events • students by weight, height, age, etc. • finding a page in a book, using a dictionary, telephone book, etc. • "Top 40" chart	• reading and interpreting tables • metric measurement • computation • averaging • number line	• newspapers • magazines • timetables • four books in *Aftermath Series*
e) Ranking things as 1st, 2nd, 3rd, etc.; smallest to largest, largest to smallest	• sports scores (track events, bowling, etc.) • ranking in class; e.g., arrival time	• graphs • estimation • standard units for age, time, etc. • collecting data	• *Guinness Book of Records* • *Almanac*
f) Coding: simple codes; identifying parts of a code	• combination lock (2-number combinations); e.g., 62 - 27 • apartment numbers, street numbers • highways • telephone numbers; e.g., 1 - 613 - 266 - 6612 — external - area - exchange - line • seating codes at stadiums; e.g. 13 - A - 9 — section - row - seat • postal code; e.g., M9C 4W2	• patterns • distinguishing the use of digits in codes and in place value numerals; e.g. room 615 • computer coding	• details from post office • student timetable

TOPICS	CONTEXTS IN WHICH TOPICS MAY BE DEVELOPED	RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
SECTION 1 NON-COMPUTATIONAL USE OF NUMBERS (Cont.)			
coding and decoding	• coded puzzles to practise computational skills • games • reflected word codes; e.g., ƎNAJUBMA	• adding and multiplying • reflection	• Books: *Mathematics, A Human Endeavor; Mathimagination*
SECTION 2 COMPUTATIONAL USE OF WHOLE NUMBERS			
a) Place value: units, tens, hundreds, thousands b) Adding pairs of numerals with up to 3 digits c) Subtracting pairs of numerals with up to 2 digits d) Multiplication facts; multiplying 1- and 2-digit numerals by 1-digit numerals e) Factoring numerals; e.g., 6 = 2 x 3 f) Dividing by 2, 3, 4, 5, 6, and 10 g) Properties of addition and multiplication, developed informally h) Reasonableness of answers	a) to g) • calculator • clock • calendar • road maps • school textbooks • games and puzzles • measuring • school population	a) to h) • counting • comparing numbers; How much larger? smaller? • increasing and decreasing • chart reading • patterns • simple formulae • metric units • flow-charting • problem solving	a) to h) • mechanical counters for money • calculator • games (bingo, etc.) • textbooks • metric ruler • abacus • golf counter • books, etc. in the catalogues from school suppliers
SECTION 3 COMPUTATIONAL USE OF DECIMALS			
a) Place value from hundreds to hundredths b) Adding and subtracting pairs of decimals of the types in a) c) Multiplying and dividing by 10, 100, 1000 d) Reasonableness of answers e) Approximating	a) to e) • using a calculator • money: writing currency; making change; adding money; "restaurant math" (menu, bill); "shopping math" (pricing, bills); wages • newspaper • textbooks • metrication • games • problem solving Note: Activities should include some whole numbers intermixed with decimals.	a) to e) • computation with whole numbers • concepts relating fractions and decimals 1/2 = 0.50 1/4 = 0.25 • estimating • approximating • number patterns • place value	a) to e) • calculator • currency • newspaper • textbooks • games • books, etc. in the catalogues of school suppliers • Appendix I, Level 3, resource materials • shopping calculators • abacus • Diennes blocks, other place value materials

STRAND NUMERICAL METHODS

TOPICS	CONTEXTS IN WHICH TOPICS MAY BE DEVELOPED	RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
SECTION 4 PERCENT			
a) Percent as hundredths; e.g., $45\% = \frac{45}{100} = 0.45$; application to finding percent of a number in real life situations	• calculator • shopping math (sales, etc.) • sales tax • newspaper, magazines, T.V., advertisements • textbooks • games • applications in school; e.g., 36% of school population is female	• adding and subtracting • decimals • comparing numbers • fractions 1/2 = 50%, etc. • collecting data	• calculator • newspaper, T.V., etc. • textbooks
SECTION 5 AIDS TO SOLVING MATHEMATICAL PROBLEMS			
a) Using simple flow charts	• flow charts of simple mathematical procedures • development of other topics in this course	• computational use of numbers • algorithms • collecting information	• flow chart template
b) Using formulae: mathematics as a language; common symbols; simple sentences; simple formulae	• formulae from the shop, science class, measurement	• perimeter, area, volume • geometry	• textbooks
c) Using a calculator	• "money math" • "supermarket math" • games Note: These aids to solving mathematical problems should be used throughout the course in developing the other topics, and will serve as an informal introduction to general problem-solving skills to be developed in future courses.	• computation • substitution in formulae • algorithms	• calculator • games
SECTION 6 FRACTIONS			
a) Fraction as part of a whole, as part of a group b) Adding and subtracting fractions with the same denominators c) Multiplying a whole number by a fraction, a fraction by a fraction d) Decimal equivalents of fractions such as 1/2, 1/5, 1/4, and 1/10	a) to d) • halves, quarters, eighths, thirds, fifths, tenths • paper folding • games • measurement • metric ruler • "shopping math" (sales, etc.) • puzzles • shop mathematics • classroom situations • calculator • money	a) to d) • computational use of whole numbers • measuring • units • mixed numbers	a) to d) • calculator • currency • ruler • games • puzzles

STRAND NUMERICAL METHODS

TOPICS	CONTEXTS IN WHICH TOPICS MAY BE DEVELOPED	RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
SECTION 7 DATA GRAPHS			
a) Reading, comparing, and interpreting information from charts and graphs b) Collecting and organizing data c) Representing data by pictographs and bar graphs	• newspapers, magazines, etc. • textbooks (geography, science, shop, etc.) • T.V. • advertising • travel • sports • timetable b), c) • data on school population, class • data from shop activities • data from community activities • data from school activities	a) to c) • counting • computational use of numbers • idea of larger, smaller • units/hour, metres, etc. • Cartesian plane • percent	a) to c) • newspapers • magazines • textbooks • T.V. • timetable
SECTION 8 MEASUREMENT			
a) Informal measuring of length, area, volume, capacity, and mass using non-standard units	• measuring objects found in the classroom • measuring objects outdoors • personal measurements • informal comparisons of length, mass, etc.	a) to h) • preservation of length, area, etc. • slides, turns, flips • tiling the plane • stacking space • decimals, fractions • place value • approximation • rounding • estimating • comparison • ratio • 3-D figures, etc. • geometric plane figures • perimeter • addition of length	• tiles • rectangular cards • ribbons • sticks (popsicle, etc.) • metal washers • bricks • trundle wheel • boxes, packages • paper • chain of paper clips • spring balance • equal arm balance
b) Measuring length, estimating	• width, height, thickness, depth, distance across • using symbols cm, m, km, imperial units (only when appropriate) • measuring objects in classroom, the classroom itself, playing field, etc. • measuring in shops, other subjects • building projects • maps, trips • sports		• metre stick • trundle wheel • measuring tape • string
c) Measuring area, estimating	• using symbols cm^2, m^2, ha, imperial units (only when appropriate) • amount of surface • painting, covering objects, carpeting, sewing, etc. • measuring in classrooms and shops • stores, supermarkets, playing field, sports • floor area of house, apartment, school	• computation • formulae • nets of 3-D figures • surface area	• metre sticks • metric paper (various sizes) • centimetre cubes • boxes (surface area)
d) Measuring volume, estimating	• using symbols cm^3, m^3, imperial units (only when appropriate) • measuring in classrooms, shops, stores, homes, etc. • volume of regular solids, irregular solids	• computation • formulae • displacement of water	• metre sticks • metric cube • centimetre cubes • boxes

STRAND NUMERICAL METHODS

TOPICS	CONTEXTS TO WHICH TOPICS MAY BE DEVELOPED	RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
SECTION 8 MEASUREMENT (Cont.)			
e) Measuring capacity, estimating	• using symbols mL, L, imperial units (only when appropriate) • using containers, boxes, etc. • measuring in classrooms, shops, homes, stores, etc.		• containers, boxes • graduated cylinder • milk cartons
f) Measuring mass, estimating	• using symbols g, kg, imperial units (only when appropriate) • measuring objects in classrooms, shops, stores, homes, etc. • mass of students, etc. • supermarket shopping		• scales • equal arm balance • postage rates • postal scale
g) Measuring temperature, estimating	• daily weather reports, forecasts • body temperature, water temperature • reading temperature from thermometer	• line graphs of temperatures vs. time	• Celsius thermometer
h) Selecting the appropriate unit; equivalent units and conversion	• observing the unit used in measuring commercial products • equivalent units, e.g. 100 cm = 1 m, etc. • converting to appropriate units; e.g., 256 cm = 2.56 m	• equivalent fractions • computation	
SECTION 9 PROBABILITY			
a) Counting the outcomes of experiments b) Formula for simple probability	a) and b) • dice • playing cards • timetable • lottery • weather • roulette wheel • coins • genetics • clothing combinations	a) and b) • graphs • computation • tables • collecting data	a) and b) • dice • cards • roulette wheel • games • coins • Hextat

TOPICS	CONTEXTS IN WHICH TOPICS MAY BE DEVELOPED	RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
SECTION 1 MOTION GEOMETRY			
Slides: drawing a slide image on a grid; identifying slide images on grids and in the real world	• tracing figures on a grid with a template • awareness of slides in the real world; e.g., streetcars • art work and lettering in ads, in magazines, on billboards • signs on buildings • machinery in shops (hoist, etc.) • movie picture frames • rows of lockers • logos • etc.	a) to c) • geometric shapes and figures • measurements • geometric constructions	a) to c) • tracing paper • grid or dot paper • geoboard • mirror • transparent mirror • paper folding
Flips: drawing a flip image on a grid; identifying flip images on grids and in the real world; symmetry	• tracing figures on a grid with a template • awareness of flips in real world; e.g., images in a mirror • lettering through glass • art work • shop machinery and activities • etc.		
Turns: 1/4, 1/2, 3/4, and full turns; drawing a turn image on a grid; identifying turn images on grids and in the real world; symmetry	• tracing figures on a grid using a template • awareness of turns in real world; e.g., streetcars, buses, cars • turning (diagram in driving manual) • machinery (lathe) • gymnastics drawings • etc.		

STRAND GEOMETRY

TOPICS	CONTEXTS IN WHICH TOPICS MAY BE DEVELOPED	RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
SECTION 2 GEOMETRIC CONSTRUCTIONS			
a) Angles: using a protractor to measure and to construct angles such as 90°, 45°, 60°, 30°; estimating the size of angles b) Constructing circles using a compass; centre, radius, diameter c) Constructing triangles using a variety of techniques; right angled, isosceles equilateral, scalene d) Regular Polygons: using a compass and protractor to construct a square, equilateral triangle, and other regular polygons in a circle; symmetry e) Tiling patterns; tiling with triangles, quadrilaterals, regular hexagons	• textbooks • paper folding • magazines • geoboard • shops • sports • clock • shops • geoboard • classroom, school • traffic signs • surface areas in classroom, school, shops • pictures in magazines, newspapers • tiling patterns in environment • art patterns • wallpaper • clothing • stacking in space	• classroom, shop, school, home • clock • art • billboards • classroom, school • advertising • jewellery • art • in nature • advertising • sports • in nature • T.V. • art • advertising	a) to d) • units • symmetry • measuring • metric measure • perimeter • area • symmetry • slides, turns, flips • geometric properties	a) to d) • protractor • compass • ruler • tracing paper • T square • set square • geoboard • transparent mirror • paper folding • clock • magazines • tiles of various shapes • Escher diagrams
SECTION 3 PRISMS AND PYRAMIDS			
a) Prisms: identified by the base; vertex, edge, face; description of prisms b) Pyramids: identified by the base; vertex, edge, face; description of pyramids	a), b) • curve stitching • points, lines, planes in classroom, shops, school, home • 3 - dimensional models • 3 - dimensional objects in classroom, school shops, home • supermarket • architecture • nets of 3-D objects	a), b) • using a compass • using a protractor • computational use of numbers • simple fractions • area, perimeter, volume • nesting objects in space • skeletons, shells, solids • point, line, plane, space • cube, cuboid, ("shoe box") • tetrahedron	a), b) • magazines • curve stitching • nets for 3-D objects • commercial kits of solids, and for building solids • toothpicks and mini-marshmallows • etc.

SUGGESTED REFERENCES

Dale Seymour et al, *Aftermath Series* (Setsco Educational Ltd.)
Steve and Janis Mercys, *Mathimagination* (Setsco Educational Ltd.)

COURSE B

for

LEVEL 2 GRADE 9 MATHEMATICS

LEVEL 1 GRADE 10 MATHEMATICS

NOTES FOR TEACHERS

TOPICS

CONTEXTS IN WHICH THE TOPICS MAY BE DEVELOPED

RELATED TOPICS, CONCEPTS, SKILLS

MATERIALS AND RESOURCES

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NOTE 2. *In designing courses at the local level, topics within the dashed rectangles [ ] are to be considered optional.*

September 1977

STRAND NUMERICAL METHODS

SECTION 1 NON-COMPUTATIONAL USE OF NUMBERS

TOPICS	CONTEXTS IN WHICH TOPICS MAY BE DEVELOPED	RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
a) Counting from 1 to 1000 by 1, 2, 5, 10, 20, 25	Counting should be handled using activities that are appropriate to the age and social maturity of the students. The following are examples of counting activities relevant to the students' experiences: • counting the number of desks in the class, cars in the parking lot, seats in the auditorium, number of boys, number of girls, number of friends; numbers on a calendar, days, weeks, months; hours, minutes, seconds; money; making change; 24-hour clock; stock in classroom, school store, supermarkets; measuring; games; population counts; probability experiments • counting on a calculator • music (Count Basie, Lawrence Welk); rhythm; number of bars, bars of rest; etc.	• comparing numbers: idea of larger, smaller • range of number sequences; largest, smallest, between • ratio • units • addition, subtraction; How much larger? smaller? • idea of before, after, between	• calendar • clock • calculator • games • Hextat • Monopoly and other money games • probability games • other games • mechanical counters
b) Writing numbers from 1 to 1000 using numerals, words	• writing cheques, legal documents • shops, other subjects • magic squares, games	• special names for numbers; e.g. pair, dozen, century, etc. • reading, spelling	• newspaper • textbooks • magazines • magic squares
c) Reading numerals	• numbers in the newspaper, magazines, on T.V., in textbooks, from other subjects (geography, science, shops, etc.) • number games • population of cities, towns, etc. • catalogues • historical dates	• place value • idea of base ten	• newspaper • T.V. • textbooks • *Almanac* • bingo • *Guinness Book of Records*
d) Arranging numbers in order, smallest to largest, largest to smallest, using numbers to sequence things	d) and e) • house numbers, locker numbers, classroom numbers, apartment numbers, parking lot spaces, calendar • reading rows and columns on student timetables • sports data (newspaper, magazines, etc.); school sports; sports records; scoring sporting events • students by weight, height, age, etc. • finding a page in a book, hymn in a hymnal; using a dictionary, telephone book, etc. • currency • "Top 40" chart	d) and e) • reading and interpreting tables, charts • metric measurement • computational use of numbers • number line • averaging • graphs • estimation • standard units • collecting data	d) and e) • newspapers • magazines • timetables • four books in *Aftermath Series* • *Guinness Book of Records* • *Almanac* • floor plan of school • money
e) Ranking things as 1st, 2nd, 3rd, etc.	• sports scores (track events, bowling, etc.) • ranking in class; e.g., arrival time • grading on report		

TOPICS	CONTEXTS IN WHICH TOPICS MAY BE DEVELOPED	RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
SECTION 1 NON-COMPUTATIONAL USE OF NUMBERS (Cont.)			
f) Coding: simple codes; identifying parts of a code;	• combination lock (2-number combinations); e.g., 62 - 27 • apartment numbers, street numbers • highways • telephone numbers; e.g., 1 - 613 - 266 - 6612 — external - area - exchange - line • seating codes at stadiums; e.g., 13 - A - 9 — section - row - seat • social insurance number • catalogue • games and puzzles • postal code; e.g., M9C 4W2 • course coding (student timetables) • supermarket	• patterns • distinguishing the use of digits in codes and in place value numerals; e.g., room 615 versus 615 dollars • filling in application forms; e.g., job application, application for social insurance number, etc. • computer coding • flow charting • reflection	• telephone book • maps • personal identification forms; e.g. driver's licence, social insurance card, credit cards • postal code directory • books: *Mathematics, A Human Endeavor; Mathimagination* • student timetable
coding and decoding	• coded puzzles to practice computational skills • games • reflected word codes; e.g., ƎNAJUBMA		
SECTION 2 COMPUTATIONAL USE OF WHOLE NUMBERS			
a) Place value: units, tens, hundreds, thousands b) Adding pairs of numerals with up to 4 digits c) Subtracting pairs of numerals with up to 4 digits d) Multiplying 1- to 3- digit numerals by 1- and 2-digit numerals; squaring numbers 1, 2, 3, . . . , 10 e) Factoring 1-and-2-digit numerals; perfect squares f) Dividing by 1- and 2-digit numerals g) Properties of addition and multiplication, developed informally	a) to j) • calculator • clock, 24-hour clock • calendar • road maps • school textbooks • games and puzzles • measuring • school population • problems in courses other than mathematics • shop mathematics	a) to j) • counting • comparing numbers; How much larger? smaller? • increasing and decreasing • chart reading • patterns • simple formulae • metric units • flow-charting • problem solving • clock arithmetics • base ten	a) to j) • calculator • clock, 24-hour clock • calendar • games (bingo, etc.) • textbooks • metric ruler • newspapers, magazines • T.V. • maps • abacus • golf score counter • see reference books, activities, etc. in catalogues from school suppliers • Diennes blocks, other place value materials
h) Reasonableness of answers i) Approximating j) Estimating	• deciding if decimal point is placed properly; e.g., 10 new cars cost $4200 or $42 000 • quantity purchase estimates; e.g., about how much should a case of coke (24 bottles) cost?		

STRAND NUMERICAL METHODS

TOPICS	CONTEXTS IN WHICH TOPICS MAY BE DEVELOPED	RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
SECTION 3 COMPUTATIONAL USE OF DECIMALS			
a) Place value from thousands to thousandths b) Adding and subtracting decimals of the type in a) c) Multiplying pairs of decimals of the type in a) d) Dividing a decimal of the type in a) by a 1-digit divisor e) Multiplying and dividing a decimal of a type in a) by 10, 100, and 1000 f) Reasonableness of answers g) Approximating h) Estimating	a) to h) • calculator • money: writing currency; making change; adding money; "restaurant math" (menu, bill); "shopping math" (pricing, bills); wages • newspaper, magazines • textbooks • metrication • problem solving • games • T.V. • "bicycle math" • "snowmobile math" • "service station math" • "plaza math" • comparative shopping, unit pricing • "travel math" • value of Canadian dollar in other currencies • etc. Note: Activities should include some whole numbers intermixed with decimals.	a) to h) • computation with whole numbers • concepts relating fractions and decimals 1/2 = 0.50 1/4 = 0.25 • estimating • patterns • place value • percentage • approximating • ratio • rates • measuring • units • reading charts • problem solving • comparing numbers • decision-making • powers of ten • informal scientific notation; e.g., 1.5 million, 10.5 mg	a) to h) • calculator • currency • newspapers, etc. • textbooks • games • T.V. • bills from restaurants and stores • see reference books, activities, etc., in catalogues of school suppliers • Appendix 1, Level 3, Grade 9 Mathematics • shopping calculators • abacus
SECTION 4 PERCENT			
a) Meaning of percent b) Applications of finding a percent of a number c) Applications of finding the percent of a part to the whole	• using a calculator • shopping math (sales, etc.) • sales tax • simple interest • newspaper, magazines, T.V., advertisements • textbooks • games • applications in school; e.g., 36% of school population is female, school grades, etc. • weather (90% chance of rain) • bonus coupons	• adding and subtracting • multiplication and division • decimals • comparing numbers • fractions 1/2 = 50%, etc. • collecting data • problem solving • probability; percent chance	• calculator • newspaper, T.V., etc. • textbooks • advertisements • centimetre cubes

TOPICS	CONTEXTS IN WHICH TOPICS MAY BE DEVELOPED	RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
SECTION 5 AIDS TO SOLVING MATHEMATICAL PROBLEMS			
a) Using simple flow charts	• flow charts of simple mathematical procedures • development of other topics in this course	• computational use of numbers • algorithms • collecting information	• flow chart template
b) Using formulae: mathematics as a language; common symbols; simple sentences; simple formulae	• formulae from the shop, science class, measurement	• perimeter, area, volume • geometry	• textbooks
c) Using a calculator	• "money math" • "supermarket math" • games Note: These aids to solving mathematical problems should be used throughout the course in developing the other topics, and will serve as an informal introduction to general problem-solving skills to be developed in future courses.	• computation • substitution in formulae • algorithms	• calculator • games
SECTION 6 FRACTIONS			
a) Fraction as part of a whole, as part of a group	• halves, quarters, eighths, thirds, fifths, tenths • paper folding • games • measurement • metric ruler • "shopping math" (sales, etc.)	a) to e) • computational use of whole numbers • measuring • units • mixed numbers • probability • data graphs	a) to e) • calculator • currency • ruler • games • puzzles • mechanical counter
b) Equivalent fractions, simple cases only	• 1/2 = 2/4 = 4/8; 3/4 = 6/8; 1/5 = 2/10, etc. • cost sharing • supermarket shopping patterns, etc. • data collection		
c) Adding and subtracting fractions with the same denominators	c) to e) • games, puzzles • shop mathematics, classroom situations • measurement • calculator • money		
d) Multiplying a whole number by a fraction, a fraction by a fraction	• shop mathematics		
e) Decimal equivalents of fractions such as 1/2, 2/5, 3/4 and 8/10	• pooling tips as a waitress or waiter; e.g. $43 among 5 people . . . 20¢ per dollar per person		

TOPICS	CONTEXTS IN WHICH TOPICS MAY BE DEVELOPED	RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
SECTION 7 DATA GRAPHS			
a) Reading, comparing, and interpreting information from charts and graphs	• newspapers, magazines, etc. • textbooks (geography, science, shop, etc.) • T.V. • advertising • travel • sports • timetable	a) to d) • counting • computational use of numbers • idea of larger, smaller • units/hour, metres, etc. • Cartesian plane • percent • scale • problem solving	a) to d) • newspapers • magazines • textbooks • T.V. • timetable • maps
b) Collecting and organizing data from real life situations, from experiments	b), c) • data on school population, class • data from shop activities • data from community activities • data from school activities		
c) Representing data by pictographs, bar graphs, and line graphs			
d) Graphing on a coordinate grid	• .maps		
SECTION 8 MEASUREMENT			
a) Informal measuring of length, area, volume, capacity, and mass using non-standard units	• measuring objects found in the classroom • measuring objects outdoors • personal measurements • informal comparisons of length, mass, etc.	a) to h) • preservation of length, area, etc. • slides, turns, flips • tiling the plane • stacking space • decimals, fractions • place value • approximation • rounding • estimating • comparison of sizes • ratio • 3-D figures, etc. • geometric plane figures • perimeter • addition of length • experiments with first class levers	• tiles • rectangular cards • ribbon • sticks (popsicle, etc.) • metal washers • bricks • trundle wheel • boxes, packages • paper • chain of paper clips • spring balance • equal arm balance
b) Measuring length, estimating	• width, height, thickness, depth, distance across • using symbols cm, m, km, imperial units (only when appropriate) • measuring objects in classroom, the classroom itself, playing field, shops, other subjects • building projects • maps, trips • sports • sizing of clothes		• metric stick • trundle wheel • mini-trundle wheel for map measuring • measuring tape • string

TOPICS	CONTEXTS IN WHICH TOPICS MAY BE DEVELOPED	RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
SECTION 8 MEASUREMENT (Cont.)			
c) Measuring area, estimating	• using symbols cm^2, m^2, ha, imperial units (only when appropriate) • amount of surface, land area • painting, covering objects, carpeting, sewing, etc. • measuring in classrooms and shops • stores, supermarkets, playing field, sports • floor area of house, apartment, school	• computation • formulae • nets of 3-D figures • surface area	• metre sticks • metric paper (various sizes) • centimetre cubes • boxes (surface area)
d) Measuring volume, estimating	• using symbols cm^3, m^3, imperial units (only when appropriate) • measuring in classrooms, shops, stores, homes, etc. • volume of regular solids, irregular solids	• computation • formulae • displacement of water	• metre sticks • metre cube • centimetre cubes
e) Measuring capacity, estimating	• using symbols mL, L, imperial units (only when appropriate) • using containers, boxes, etc. • measuring in classrooms, shops, homes, stores, etc.	• computation	• containers, boxes • graduated cylinder • milk cartons, etc.
f) Measuring mass, estimating	• using symbols g, kg, t, imperial units (only when appropriate) • measuring objects in classrooms, shops, stores, homes, etc. • finding the mass of students, etc. • supermarket shopping	• experiments with first class lever	• scales • equal arm balance • postage rates • postal scales
g) Measuring temperature, estimating	• Celsius units • daily weather reports, forecasts • body temperature, water temperature • reading temperature from thermometer	• line graphs of temperatures vs. time • probability, weather forecasts, percent • positive and negative numbers, oppositeness	• Celsius thermometer
h) Selecting the appropriate unit; equivalent units and conversion	• observing the unit used in measuring commercial products • equivalent units; e.g., 12 000 000 cm^3 = 12 m^3	• equivalent fractions • computation	
SECTION 9 PROBABILITY			
a) Counting the outcomes of experiments b) Formula for simple probability c) Finding the probability using a tree diagram	a) to c) • dice • playing cards • timetable • lottery • weather • roulette wheel • coins • genetics • clothing combinations • sports	a) to c) • graphs • computation • tables • collecting data	a) to c) • dice • cards • roulette wheel • games • coins • Hextat

STRAND GEOMETRY

TOPICS	CONTEXTS IN WHICH TOPICS MAY BE DEVELOPED	RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
SECTION 1 MOTION GEOMETRY			
a) Slides: drawing a slide image on a grid, on plain paper, identifying slide images in diagrams and in the real world	• tracing figures with a template on a grid and on plain paper • awareness of slides in the real world (e.g., cars on road, ski-lift, etc.) • art work and lettering in ads, in magazines, on billboards • signs on buildings • machinery in shops (hoist, etc.) • movie picture frames • rows of lockers • logos • etc.	a) to c) • geometric shapes and figures • measurements • geometric construction	a) to c) • tracing paper • grid or dot paper • geoboard • mirror • transparent mirror • paper folding
b) Flips: drawing a flip image on a grid, on plain paper, identifying flip images in diagrams and in the real world; symmetry	• tracing figures with a template on a grid and on plain paper • awareness of flips in real world; (e.g., images in a mirror) • lettering through glass • art work • shop machinery and activities • etc.		
c) Turns: 1/4, 1/2, 3/4, and full turns; drawing a turn image on a grid; identifying turn images in diagrams and in the real world; symmetry	• tracing figures using a template on a grid and on plain paper • awareness of turns in real world (e.g. streetcars, buses, cars) • turning (diagram in driving manual) • machinery (lathe) • gymnastics drawings • etc.		
d) Distortions: drawing a distortion image using a distorted grid	• awareness of distortions in the real world; e.g. reflection images in irregular mirrors		

TOPICS	CONTEXTS IN WHICH TOPICS MAY BE DEVELOPED		RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
SECTION 2 GEOMETRIC CONSTRUCTIONS				
a) Angles: using a protractor to measure and to construct angles from 0° to 90°; estimating the size of angles	• textbooks • paper folding • geoboard • magazines, newspapers	• classroom, shop, school, home • clock • art	a) to d) • units • symmetry • measuring • metric measure • perimeter • area	a) to d) • protractor • compass • ruler • tracing paper • T square • set square • geoboard • transparent mirror • paper folding • clock • magazines
b) Constructing circles using a compass; centre, radius, diameter	• shops • sports • clock • in nature	• classroom, school • advertising • jewellery • art		
c) Constructing triangles using a variety of techniques; right angled, isosceles, equilateral, scalene	c) and d) • shops • geoboard • classroom, school	• advertising • sports • in nature		
d) Constructing squares and rectangles using a variety of techniques				
e) Regular polygons: using a compass and protractor to construct a square, equilateral triangle, and other regular polygons in a circle: symmetry	• traffic signs • surface areas in classroom, school, shops • T.V.	• pictures in magazines, newspapers • art • advertising		
f) Tiling patterns; tiling with triangles, quadrilaterals, regular hexagons; symmetry	• tiling patterns in environment • art patterns	• wallpaper • clothing • stacking in space	• symmetry • slides, turns, flips • geometric properties	• tiles of various shapes • Escher patterns
SECTION 3 PRISMS AND PYRAMIDS				
a) Prisms: identified by the base; vertex, edge, face; description of prisms; examples of prisms in the real world b) Pyramids: identified by the base; vertex, edge, face; description of pyramids; examples of pyramids in the real world	a), b) • curve stitching • points, lines, planes in classroom, shops, school, home • 3- dimensional models • 3- dimensional objects in classroom, school, shops, homes • supermarket • architecture, buildings • nets of 3-D objects • nature		a), b) • using a compass • using a protractor • computational use of numbers • simple fractions • area, perimeter, volume • nesting objects in space • skeletons, shells, solids • polyhedra	a), b) • magazines • curve stitching • 3-D models • kits for building skeletons of prisms and pyramids • nets for 3-D objects • commercial kits of solids, and for building solids • toothpicks and mini-marshmallows, etc. • polyhedra kits

SUGGESTED REFERENCES

Dale Seymour et al, *Aftermath Series* (Setsco Educational Ltd.)
Steve and Janis Marcys, *Mathimagination* (Setsco Educational Ltd.)
Harold Jacobs, *Mathematics: A Human Endeavor* (W. H. Freeman and Co.)

COURSE C

for

LEVEL 2 GRADE 10 MATHEMATICS

LEVEL 1 GRADE 11 MATHEMATICS

NOTES FOR TEACHERS

TOPICS

CONTEXTS IN WHICH THE TOPICS MAY BE DEVELOPED

RELATED TOPICS, CONCEPTS, SKILLS

MATERIALS AND RESOURCES

NOTE 1. *This is a DRAFT COPY for validation purposes and for optional implementation during the school years 1977 – 1978 and 1978 – 1979. Reactions should be forwarded to the Intermediate Mathematics Coordinator, Ministry of Education, Curriculum Branch, Mowat Block, Queen's Park, Toronto M7A 1L2.*

NOTE 2. *In designing courses at the local level, topics within the dashed rectangles [ ] are to be considered optional.*

September 1977

STRAND NUMERICAL METHODS

TOPICS	CONTEXTS IN WHICH TOPICS MAY BE DEVELOPED	RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
SECTION 1 NON-COMPUTATIONAL USE OF NUMBERS			
a) Counting (unrestricted) by 2, 3, 4, 5, 10, 20, 25; notion of finite sequences, infinite sequences	Counting should be handled using activities that are appropriate to the age and social maturity of the students. The following are examples of counting activities relevant to the students' experiences: • counting the number of desks in the class, cars in the parking lot, seats in the auditorium, number of boys, number of girls, number of friends; numbers on a calendar, days, weeks, months; hours, minutes, seconds; money; making change; 24-hour clock; stock in classroom, school store, supermarkets; measuring; games; population counts in class, school, town, city, country; probability experiments; astronomy • counting on a calculator • music (Count Basie, Lawrence Welk); rhythm; number of bars, bars of rest; etc.	• comparing the size of numbers: • range of number sequences • computation • ratio • counting activities in geometry • units • scientific notation	• calendar • clock • calculator • games • Hextat • Monopoly and other money games • probability games • other games • mechanical counters
b) Writing numbers (unrestricted) using numerals, words	• writing cheques, legal documents • shops, other subjects • magic squares, games	• special names for numbers; e.g. pair, dozen, century, etc. • reading, spelling	• newspaper • textbooks • magazines • magic squares
c) Reading numerals	• numbers in the newspaper, magazines, on T.V., in textbooks, from other subjects (geography, science, shops, etc.) • number games • population of cities, towns, etc. • catalogues • historical dates	• place value • idea of base ten	• newspaper • T.V. • textbooks • *Almanac* • bingo, games • *Guinness Book of Records*
d) Arranging numbers in order; using numbers to sequence things	d) and e) • house numbers, street numbers, locker numbers, classroom numbers, apartment numbers, parking lot spaces, calendar • reading rows and columns on student timetables • sports data (newspaper, magazines, etc.); school sports; sports records; scoring sporting events • students by weight, height, age, etc. • finding a page in a book, hymn in a hymnal; using a dictionary, telephone book, etc. • currency • "Top 40" chart • information in newspapers, magazines, etc.	d) and e) • reading and interpreting tables, charts • metric measurement • computational use of numbers • number line • averaging • graphs • estimation • standard units for age, time, etc. • collecting data	d) and e) • newspapers • magazines • timetables • four books in *Aftermath* series • *Guinness Book of Records* • *Almanac* • floor plan of school • money
e) Ranking things as 1st, 2nd, 3rd, etc.	• sports scores (track events, bowling, etc.) • ranking in class; e.g., arrival time • grading on report		

TOPICS	CONTEXTS IN WHICH TOPICS MAY BE DEVELOPED	RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
SECTION 1 NON-COMPUTATIONAL USE OF NUMBERS (Cont.)			
f) Coding: simple codes; identifying parts of a code;	• combination locks • apartment numbers, street numbers • highways • telephone numbers; e.g. 1 - 613 - 266 - 6612 — external - area - exchange - line • seating codes at stadiums; e.g., 13 - A - 9 — section - row - seat • social insurance number • catalogues • games and puzzles • postal code; e.g., M9C 4W2 • course coding (student timetables) • supermarket	• patterns • distinguishing the use of digits in codes and in place value numerals; e.g., room 615 versus 615 dollars • filling in application forms; e.g., job application, application for social insurance number, etc. • computer coding • flow charting • adding and multiplying • reflection	• telephone book • maps • personal identification forms; e.g., driver's licence, social insurance card, credit cards • postal code directory • books: *Mathematics, A Human Endeavor; Mathimagination* • student timetables • *Cardiac* (simulation game, Bell Telephone) • birth certificate
coding and decoding	• coded puzzles to practise computational skills • games • reflected word codes; e.g. AMBULANCE • numerical — literal codes; key word codes, etc. • Morse code • Semaphore codes (flags, etc.)		
SECTION 2 COMPUTATIONAL USE OF WHOLE NUMBERS			
a) Place value b) Adding and subtracting numerals with up to 4 digits, in columns and in rows c) Multiplying 1- to 4-digit numerals; squaring numbers 1, 2, 3, . . . , 10; cubing numbers 1, 2, 3, 4 d) Factoring 1- and 2-digit numerals; perfect squares and their square roots e) Dividing by 1- to 3-digit numerals f) Properties of addition and multiplication, developed informally g) Reasonableness of answers h) Approximating i) Estimating	a) to i) • calculator • clock, 24-hour clock • calendar • road maps • school textbooks • games and puzzles • measuring • "school mathematics", e.g., school population, related statistics • problems in courses other than mathematics • shop mathematics • newspapers, magazines • T.V. • using road maps, • number patterns • sports • deciding if decimal point is placed properly; e.g. 10 new cars cost $4200 or $42000 • quantity purchase estimates; e.g., about how much should a case of coke (24 bottles) cost?	a) to i) • counting • comparing numbers; How much larger? smaller? • increasing and decreasing • reading tables • patterns • simple formulae • metric units • flow-charting • problem solving • clock arithmetics • base ten • reading forms	a) to i) • calculator • clock, 24-hour clock • calendar • games (bingo, etc.) • textbooks • metric ruler • newspapers, magazines • T.V. • forms; e.g. application forms • maps • abacus • golf score counter • see reference books, activities etc. in catalogues from school suppliers • Diennes blocks, other place value materials

SECTION 3 COMPUTATIONAL USE OF DECIMALS

TOPICS	CONTEXTS IN WHICH TOPICS MAY BE DEVELOPED	RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
a) Place value (unrestricted) b) Adding and subtracting decimals in columns and in rows c) Multiplying decimals d) Dividing decimals by 1- to 3-digit numerals e) Multiplying and dividing a decimal by 10, 100, and 1000, etc., and by 0.1, 0.01, etc. f) Reasonableness of answers g) Approximating h) Estimating i) Rounding and truncating	a) to i) • calculator • money: writing currency; making change; adding money; "restaurant math" (menu, bill); "shopping math" (pricing, bills); wages • newspaper, magazines • textbooks • metrication • problem solving • games • T.V. • "bicycle math" • "snowmobile math" • "service station math" • "plaza math" • "travel math" • comparative shopping, unit pricing • value of Canadian dollar in other currencies • budgeting • part-time job (wages) Note: Activities should include some whole numbers intermixed with decimals.	a) to i) • computation with whole numbers • concepts relating fractions and decimals • estimating • approximating • patterns • place value • percentage • ratio • rates • measuring • units • reading charts • problem solving • comparing numbers • decision-making • powers of ten • informal scientific notation; e.g., 1.5 million, 10.5 mg	a) to i) • calculator • currency • newspapers • textbooks • games • T.V. • bills from restaurants and stores • see reference books, activities, etc., in catalogues of school suppliers • Appendix 1 of Level 3 resource materials • shopping calculators • abacus

SECTION 4 PERCENT

a) Meaning of percent b) Applications of finding a percent of a number c) Applications of finding the percent of a part to the whole	a) to c) • using a calculator • shopping math (sales, etc.) • sales tax • simple interest, savings accounts, bonds, etc. • newspapers, magazines, T.V., advertisements • textbooks • games • applications in school; e.g., 36% of school population is female, school grades, etc. • discounts, "money off" coupons, money coupons • commission; percent of sales • weather (90% chance of rain) • bonus coupons	• computation • decimals • comparing numbers • fractions 1/2 = 50%, etc. • collecting data • problem solving • probability; percent chance	• calculator • newspaper, T.V., etc. • textbooks • advertisements • centimetre cubes • games

TOPICS	CONTEXTS IN WHICH TOPICS MAY BE DEVELOPED	RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
SECTION 5 PROBLEM SOLVING			
a) Solving a variety of real world and mathematical problems by a method such as:			• Teacher reference: *Inquiry Training, Fusing Theory and Practice* (O.I.S.E.)
• identifying the problem	• What is the problem? What are you trying to do?		
• collecting and organizing the significant information	• restating problem in own words • collecting additional information related to the problem • listing information to be used • understanding the problem • drawing a diagram, when appropriate		
• analysing the information and developing a strategy	• representing the procedure by a simple flow chart • using a formula (when appropriate) – the symbolic language of mathematics – common symbols; e.g., x, +, = – simple formulae as mathematical sentences for commonly occurring situations; e.g., A = lw for area of a rectangle – formulae from shops, science, measurement, etc. • solving a simpler, but similar problem	• algorithms	
• executing the strategy and testing the reasonableness of the result	• aids to working out a solution: – calculator – computer (informal discussion of how a computer works, on - off, memory, language, programming) – substituting in a formula – computation • testing the answer with a calculator, or otherwise	• computation	• calculator
• presenting the solution if the result is reasonable, or repeating the above	• write out the solution, if the result seems reasonable; otherwise, repeat the procedure Sources of problems: mathematics, other subjects, shops, social planning, supermarket, money matters, measurement, projects, games, recreation, sports, Cardiac, daily activities, school activities, social activities, community activities Nature of problems: straightforward, simple skills, include choosing alternatives		• games • *Cardiac*

TOPICS	CONTEXTS IN WHICH TOPICS MAY BE DEVELOPED	RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
SECTION 6 FRACTIONS			
a) Fraction as part of a whole, as part of a group	• halves, quarters, eighths, thirds, fifths, tenths • paper folding • games • measurement • metric ruler • "shopping math" (sales, etc.)	a) to e) • computational use of whole numbers • measuring • units • mixed numbers • probability	a) to e) • calculator • currency • ruler • games • puzzles • mechanical counter
b) Equivalent fractions: reduction of fractions (simple cases only)	• 1/2 = 2/4 = 4/8; 3/4 = 6/8; 1/5 = 2/10, etc. • cost sharing • supermarket shopping patterns, etc. • data collection		
c) Adding and subtracting fractions with the same denominators, with different denominators	c) to e) • games, puzzles • shop mathematics, classroom situations • measurement • calculator • money		
d) Multiplying a whole number by a fraction, a fraction by a fraction	• shop mathematics		
e) Decimal equivalents of fractions such as 1/2, 2/5, 3/4 and 1/3	• pooling tips as a waitress or waiter; e.g. $43 among 5 people . . . 20¢ per dollar per person		
SECTION 7 DATA GRAPHS			
a) Reading, comparing, and interpreting information from charts, graphs, and tables	• newspapers, magazines, etc. • textbooks (geography, science, shop, etc.) • T.V. • advertising • travel • sports • timetable	a) to e) • counting • computational use of numbers • idea of larger, smaller • units/hour, metres, etc. • Cartesian plane • percent • scale • problem solving • graphing on different grids (enlargements, reductions, distortions)	a) to e) • newspapers • magazines • textbooks • T.V. • timetable • maps
b) Collecting and organizing data from real life situations, from experiments; polling, surveying, sampling	b) to e) • data on school population, class • data from shop activities • data from community activities • data from school activities		
c) Representing data by pictographs, bar graphs, circle graphs, and line graphs			
d) Representing data in tables; mean, median, and mode of the data			
e) Graphing on a coordinate grid	• maps		

TOPICS	CONTEXTS IN WHICH TOPICS MAY BE DEVELOPED	RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
SECTION 8 MEASUREMENT			
a) Measuring length, estimating	• width, height, thickness, depth, distance across • using symbols cm, m, km, imperial units (only when appropriate) • measuring objects in classroom, the classroom itself, the playing field, shops, other subjects • building projects • maps, trips • sports • sizing of clothes • personal measurements	• perimeter • addition of length • experiments with first class levers	• metre sticks • trundle wheel • measuring tape • string • maps • mini-trundle wheel for map measuring
b) Measuring area, estimating	• using symbols cm^2, m^2, ha, imperial units (only when appropriate) • amount of surface, land area • painting, covering objects, carpeting, sewing, etc. • measuring in classrooms and shops • stores, supermarkets, playing field, sports • floor area of house, apartment, school	• computation • formulae • nets of 3-D figures • surface area	• metre sticks • metric paper (various sizes) • centimetre cubes • boxes (surface area) • maps
c) Measuring volume, estimating	• using symbols cm^3, m^3, imperial units (only when appropriate) • measuring in classrooms, shops, stores, homes, etc. • volume of regular solids, irregular solids	• computation • formulae • displacement of water	• metre sticks • metre cube • centimetre cubes • boxes
d) Measuring capacity, estimating	• using symbols mL, L, imperial units (only when appropriate) • using containers, boxes, etc. • measuring in classrooms, shops, homes, stores, etc.	• computation	• containers, boxes • graduated cylinder • milk cartons, etc.
e) Measuring mass, estimating	• using symbols g, kg, t, imperial units (only when appropriate) • measuring objects in classrooms, shops, stores, homes, etc. • finding the mass of students, etc. • supermarket shopping	• experiments with first class lever	• scales • equal arm balance • postage rates • postal scales
f) Measuring temperature, estimating	• Celsius units • daily weather reports, forecasts • body temperature, water temperature • reading temperature from thermometer	• line graphs of temperatures vs. time • probability, weather forecasts, percent • positive and negative numbers, oppositeness • heat versus temperature • calories	• Celsius thermometer
g) Measuring pressure	• using symbol kPa, and imperial units (only when appropriate) • air pressure in a balloon, an air mattress, inflatable furniture, etc. • air pressure in a tire (bicycle, car, etc.) • atmospheric pressure, water pressure, liquid pressure • boiling point of water at sea level, at a high altitude, and when contained under pressure • oil pressure in a car (engine, shock absorbers, braking system) • pressure in radiator of car • expansion and contraction with respect to heat • blimps and dirigibles • lift on airplane wings	• boiling point of water • units (metric, imperial) • conversion of units	• air pressure gauge • air pump • barometer • bicycle wheel and tire • balloon • toys propelled by air pressure of a balloon • sink plunger • suction cup • paper gliders, model airplane

STRAND NUMERICAL METHODS

TOPICS	CONTEXTS IN WHICH TOPICS MAY BE DEVELOPED	RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
SECTION 8 MEASUREMENT (Cont.)			
h) Selecting the appropriate unit; equivalent units and conversion	• observing the unit used in measuring commercial products • equivalent units; e.g., 1 200 000 cm^3 = 1.2 m^3	• equivalent fractions	
SECTION 9 PROBABILITY			
a) Counting the outcomes of experiments b) Formula for simple probability c) Finding the probability using a tree diagram d) The Fundamental Counting Principle; applications e) Odds f) Dependent events, independent events	a) to e) • dice • playing cards • timetable • lottery • weather • roulette wheel • coins • genetics • clothing combinations • sports • codes • bingo • social insurance numbers • licence plates • telephone numbers • sports	a) to e) • graphs • computation • tables • collecting data	a) to e) • dice • cards • roulette wheel • games • coins • Hextat

TOPICS	CONTEXTS IN WHICH TOPICS MAY BE DEVELOPED	RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
SECTION 1 MOTION GEOMETRY			
a) Slides: drawing a slide image on a grid, on plain paper; identifying slide images in diagrams and in the real world	• tracing figures with a template on a grid, and on plain paper • awareness of slides in the real world (e.g., cars on road, ski-lift, etc.) • art work and lettering in ads, in magazines, on billboards • signs on buildings • machinery in shops (hoist, etc.) • movie picture frames • rows of lockers • logos • etc.	a) to c) • geometric shapes and figures • measurements • geometric construction	a) to c) • tracing paper • grid or dot paper • geoboard • mirror • transparent mirror • paper folding
b) Flips: drawing a flip image on a grid, on plain paper; identifying flip images in diagrams and in the real world; symmetry	• tracing figures with a template on a grid, and on plain paper • awareness of flips in the real world (e.g., images in a mirror) • awareness of flips in real world • lettering through glass • art work • shop machinery and activities • etc.		
c) Turns: drawing a turn image on a grid, on plain paper; identifying turn images in diagrams and in the real world; symmetry	• tracing figures using a template on a grid and on plain paper • awareness of turns in real world (e.g. streetcars, buses, cars) • turning (diagram in driving manual) • machinery (lathe) • gymnastics drawings • etc.		
d) Enlargements, Reductions: drawing an enlarged or reduced image on a grid; size and shape; identifying enlarged or reduced images in diagrams and in the real world; scale drawings	• photography • maps • scale drawings of floor plans, furniture, etc. • microfiche	• multiplication, division • ratio • scale • similar figures; size, shape	• regular grids of different sizes • designs • maps • floor plans • scale drawings
e) Distortions: drawing a distortion image using a distorted grid	• awareness of distortions in the real world; e.g. reflection images in irregular mirrors		• various mirrors: concave, convex, cylindrical, irregular • grids: regular and distorted • designs superimposed on the grids
SECTION 2 GEOMETRIC CONSTRUCTIONS			
a) Angles: using a protractor to measure and to construct angles from 0° to 180°; estimating the size of angles	• textbooks • paper folding • geoboard • magazines, newspapers • classroom, shop, school, home • clock • art	a) to d) • units • symmetry • measuring • metric measure • perimeter	a) to d) • protractor • compass • ruler • tracing paper • T square

STRAND GEOMETRY

TOPICS	CONTEXTS IN WHICH TOPICS MAY BE DEVELOPED	RELATED TOPICS, CONCEPTS, SKILLS	MATERIALS AND RESOURCES
SECTION 2 GEOMETRIC CONSTRUCTIONS (Cont.)			
b) Constructing circles using a compass; centre, radius, diameter	• shops • sports • clock • in nature • classroom, school • advertising • jewellery • art	• area	• set square • geoboard • transparent mirror • paper folding • clock • magazines
c) Constructing triangles using a variety of techniques; right angled, isosceles, equilateral, scalene d) Constructing squares and rectangles using a variety of techniques	c) and d) • shops • geoboard • classroom, school • advertising • sports • in nature		
e) Regular polygons: using a compass and protractor to construct a square, equilateral triangle, and other regular polygons in a circle; symmetry	• traffic signs • surface areas in classroom, school, shops • T.V. • pictures in magazines, newspapers • art • advertising		
f) Tiling patterns: tiling with triangles, quadrilaterals, regular hexagons; geometric properties; mosaics; symmetry	• tiling patterns in environment • art patterns • wallpaper • clothing • stacking in space	• symmetry • slides, turns, flips • geometric properties	• tiles of various shapes • Escher patterns
SECTION 3 PRISMS AND PYRAMIDS			
a) Prisms: identified by the base; vertex, edge, face; characteristics of prisms; examples of prisms in the real world; right prisms and symmetry b) Pyramids: identified by the base; vertex, edge, face; description of pyramids; examples of pyramids in the real world; right pyramids and symmetry c) Constructing prisms and pyramids as shells, as skeletons d) Stacking space	a) to d) • curve stitching • points, lines, planes in classroom, shops, school, home • 3-dimensional models • 3-dimensional objects in classroom, school, shops, homes • supermarket • architecture, buildings • nets of 3-D objects • nature	a) to d) • using a compass • using a protractor • computational use of numbers • simple fractions • area, perimeter, volume • nesting objects in space • skeletons, shells, solids • polyhedra	a) to d) • magazines • curve stitching • 3-D models • kits for building skeletons of prisms and pyramids • nets for 3-D objects • commercial kits of solids, and for building solids • toothpicks and mini-marshmallows, etc. • polyhedra kits

SUGGESTED REFERENCES

Dale Seymour et al, *Aftermath Series* (Setsco Educational Ltd.)
Steve and Janis Marcys, *Mathimagination* (Setsco Educational Ltd.)

LEVEL 3 GRADE 9 MATHEMATICS (BASIC)

NOTES FOR TEACHERS

TOPICS

SUGGESTED DEVELOPMENT

RELATED TOPICS, CONCEPTS, SKILLS

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September 1977

LEVEL 3 GRADE 9 MATHEMATICS (BASIC)

NOTES FOR TEACHERS

Developing a Program Locally

The core topics should be mastered in order to provide a solid basis for further study. This background will enable some of the students, upon completion of this course, to pursue Level 4, Grade 9 Mathematics (General) in the following year. This course also provides the skill base for students who go on to Level 3, Grade 10 Mathematics (Basic). In addition to the core topics, some of the options should be explored to provide better balance to the program.

Teaching order is not implied by the numbering of the sections and subsections. In fact, integrating topics from several of these sections around a theme will provide excellent motivation — see Appendices I and II.

Timing of the program and the skill base for beginning the study of each section should be based on local conditions and are left to the judgement of individual teachers.

LEVEL 3 GRADE 9 MATHEMATICS (BASIC)

NOTES FOR TEACHERS

:AND NUMERICAL METHODS

TOPICS	SUGGESTED DEVELOPMENT	RELATED TOPICS, CONCEPTS, SKILLS
:TION 1 THE COUNTING NUMBERS		
Counting; numerals and place value	• counting by beginning at any number and using suitable increments, counting to any number • counting, using a calculator • positional numeration systems; place value • non-standard use of place value in numerals; e.g. Apartment No. 2117	• arrangement for counting efficiency • vocabulary development; larger, smaller, successor, etc. • large numbers • the notion of finiteness • divisibility tests • rounding numbers • approximating
Reading numerals, writing numerals in words	• converting numerals to words and words to numerals	
Understanding and using special numerical terms	• terms such as pair, triplet, quartet, quintet, octet, decade, dozen, score, gross, century, millenium might be discussed • the year-century relation	
The concepts of zero and one	• zero as an extension of the counting numbers; properties of 0 and 1 • zero as a placeholder; meaning of a zero to the left and to the right of a non-zero placeholder	• powers of 10 • naming and reading beyond 10^6
Ordering and sequencing	• order as an outcome of an activity: first, second, third, etc. • ascending and descending arrangements of numbers • ordering numerical data; ranking of an element in the order	• flow diagrams • arithmetic and geometric patterns • data interpretation • chart and graph reading and construction
The notion of a code; coding and decoding	• the meaning and logic of well-known codes such as postal, student number, combination of lock, telephone numbers • constructing and using a code • codes with a check digit e.g. Book Coding: ISBN 0-673-13100-9	• use of place value • non-positional number systems • floor plan labelling • use of implied coordinates • simplification of data collection • charts and graphs • pattern recognition • informal induction
:TION 2 COMPUTATION WITH WHOLE NUMBERS AND DECIMALS		
Rounding	• given a number, rounding it to the nearest 10, 100, 1000, and so on • measurements to the nearest metre, centimetre, millimetre	• estimation, approximation, or both; checking of calculations
Addition, subtraction, multiplication, and division with whole numbers	• meaning of operations • basic facts of addition and multiplication, with application to subtraction and division • techniques for adding columns • special multiplying aids such as: by 10, 100, 1000, etc.; by 11; by 5; and other simple algorithms • simple divisibility tests • multiplying on paper with up to 2-digit by 3-digit numerals • short division with 1-digit divisor • dividing on paper with up to 5-digit by 2-digit numerals, with remainder and quotient up to one decimal place • extended calculations using a calculator	• sense of required accuracy • applications to sports, dieting, travel, income, etc. • magic squares • palindromic numbers • powers • alphametics, such as finding digits to represent the letters in a problem; for example, ONE + ONE = TWO
Calculations related to money	• sums and differences of money quantities • product of whole number and a money amount • quotients arising from "cost per item", and "number of items" • (money) x (money) products are meaningless	• unit pricing • budgeting, comparison shopping • wages • checking bills, monthly statements, etc. • formulae

STRAND NUMERICAL METHODS

TOPICS	SUGGESTED DEVELOPMENT	RELATED TOPICS, CONCEPTS, SKILLS
SECTION 2 COMPUTATION WITH WHOLE NUMBERS AND DECIMALS (Cont.)		
d) Operations with decimals	• adding money amounts with attention to position of decimal point; subtracting • adding decimals with emphasis on position of decimal point to preserve place values; subtracting • the meaning and nomenclature of the positions to the right of the decimal	• metric system • rounding and estimating • shop and science problems
SECTION 3 FRACTIONS		
a) The concept of fraction, mixed number	• recognizing and naming real world situations in which fractions arise • whole number plus a fraction less than 1 written together	• numerator and denominato • fractions which add to 1
b) Equivalent fractions	• $\frac{a}{b} = \frac{ax}{bx}$, and particularly $1 = \frac{1}{1} = \frac{x}{x}$ for $x \neq 0$	
c) Conversions and simplifications	• converting a fraction greater than 1 to a mixed number • reducing a fraction to lowest terms • converting a mixed number to fractional form	
d) Conversion of a fraction to a decimal	• converting a fraction to a decimal • recognizing a fraction in its equivalent decimal form • converting a terminating decimal to a fraction in lowest terms • algorithms for multiplying and dividing decimals • calculator for extended decimal computation	• division of whole numbers • multiplication of whole numbers
e) Multiplying and dividing with fractions and mixed numbers	• computing with fractions might be restricted to denominators such as 2, 3, 5, 8, 10 and to mixed numbers less than 10 • the idea of reciprocal fraction; computing with "reasonable" numbers • using problems from the real world, and a variety of puzzles and games to practise these skills	• ordering fractions and mix numbers • ratios • enlarging and reducing
f) Common denominator; addition and subtraction of fractions and mixed numbers		
SECTION 4 RATIO AND RATE		
a) The concept of ratio; equivalent ratios	• using a fraction to represent the comparison of two quantities; e.g., $\frac{\text{length}}{\text{width}}$ • notation; e.g., l:w • the concept of equivalent ratios, developed from the concept of equivalent fractions	• mixture problems such as: by volume, cement/sand = 1/4; find the volume of sand needed for 200 m^3 of cement. • odds
b) The concept of rate	• the concept of rate; the "per unit" idea	• allowance (per week, per d • fuel consumption
SECTION 5 PERCENT		
a) The concept of percent	• percent as hundredths; decimal form • percent as a fraction with denominator 100; fractional form	
b) Conversion of percents to decimals, decimals to percents	• converting a percent to a decimal by dividing by 100; a decimal to a percent by multiplying by 100 • converting percents to fractions with denominator 100, then reducing; fractions to percents	• construction of circle grap • scale factor as percent in scale diagrams
c) Computation with percents	• calculating a percent of a number • calculating the percent that one number is of another • applying percents to school grades, sales tax, discounts, and other useful situations	• formulae

LEVEL 3 GRADE 9 MATHEMATICS (BASIC)

NOTES FOR TEACHERS

'RAND NUMERICAL METHODS

TOPICS	SUGGESTED DEVELOPMENT	RELATED TOPICS, CONCEPTS, SKILLS
ECTION 6 DATA PRESENTATION		
Reading Tables	• reading two column tables, such as a sales tax table • reading tables with many columns, using row and column referencing • interpolating and extrapolating tabular data, when meaningful; e.g., a table graduated in 10-year intervals giving traffic accident figures could be interpolated for the figure for 1965	• computational use of numbers • problem solving
Reading pictographs	• analysing simple pictographs for "most", "least", and so on • using the scale of the pictograph to determine the approximate numbers involved, if not specifically stated • computing differences; and the percent increase or decrease; e.g., increase in C.P.I. market basket	• computation
Understanding bar, line, and circle graphs	• reading a single bar graph; multiple bar graph; line graph; multiple line graph; circle graph • comparing two quantities from a graph as a ratio, as a percent • computing percent increases and/or decreases • observing the impact of scale choice; deception using graphs	• geometry • computation
) Collecting data: attribute data,	• using a tally chart, one cell for each attribute, and then counting to form a frequency table; e.g. compare the makes of cars on a parking lot	• counting techniques
numerical data	• using a simple tally chart, as in attribute data, when only a small set of possible results may occur; frequency table; e.g., the outcomes of successive rolls of a die • tallying in intervals where a large number of possible results occurs; usually 10 — 20 intervals will suffice; frequency table, e.g. sequence of purchases at supermarket checkout	• counting • measurement • money and decimal calculations
) Drawing graphs: pictographs,	• studying the frequency table for attribute data; deciding on a suitable symbol, and value (scale) for its presentation; title	• accurate constructions • measurement
bar graphs,	• deciding a scale for the data; grid paper as a drawing aid; drawing the bar graph; title	• measurement • locating points on a grid
line graphs,	• selecting a horizontal and a vertical scale; locating points; title	• coordinate system
circle graphs	• extending a frequency table to include relative frequency ratios; relative frequency ratios expressed as percents; converting percents to the number of degrees • deciding on size of graph and subdividing into components; title • particular advantage of a circle graph in comparing a whole and its parts; e.g., the graph of one's budget	• measurement • numerical practice • geometry; use of protractor; accurate constructions
SECTION 7 POWERS AND ROOTS		
a) The meaning of power notation	• power notation as a form of shorthand for multiple factors, such as: $3 \times 3 \times 3 \times 3 \times 2 \times 2 = 3^4 2^2$ • writing powers in expanded form, expanded forms as powers • evaluation of powers with whole number exponents	• vocabulary: square, cube, exponent, base • geometric meaning of squares and cubes • Pythagorean Theorem
b) Square root as the inverse of square	• constructing the square — square root table by reversing the $n \to n^2$ table, up to $n = 15$ • the square root of a number equal to or less than 225; approximations to one decimal place	• interpolation

LEVEL 3 GRADE 9 MATHEMATICS (BASIC)

NOTES FOR TEACHERS

STRAND NUMERICAL METHODS

TOPICS	SUGGESTED DEVELOPMENT	RELATED TOPICS CONCEPTS, SKILLS
SECTION 8 INTEGERS		
a) The concept of integer	• observing the use of opposite, or integer concept, in the real world; e.g., in temperature, and sports	
b) The order of integers; the l-line	• identifying the opposite of an integer • locating the integer and its opposite on the l-line • interpretation of positive and negative signs in contexts such as travel, and gain and loss	
c) Operations with integers	• developing the skills of adding, subtracting, multiplying, and dividing integers	• formulae
SECTION 9 MEASUREMENT		
a) Metric units; suitability,	• different units that represent the same quantity, such as 1 000 000 000 mm^3 and 1 m^3 • understanding the relative sizes of units through activities which require selection of a suitable unit, such as "In what units would a person buy bananas? medicine? sugar?", and so on	• multiplication and division by powers of 10 • comparative shopping • unit pricing
conversion of units	• converting the answer to a more suitable unit — too large? too small?	
b) Length; use of metric symbols mm, cm, m, km; estimating	• using a ruler to measure the length of various segments, and the dimensions of various objects • the concept of perimeter of a closed figure as the distance along a figure, the sum of the sides • estimating then checking by measurement • reasonableness of results; for example, the length of a new pencil as 20 mm, 20 cm, or 20 m ? • accuracy of measurement related to quality of ruler; increasing the accuracy by using dividers • indirect measurement, such as using string for finding the length of a curve	• formulae: $P = 2(l + w)$ $P = s_1 + s_2 + s_3$ • open figures; closed figures • conversions of units
c) Area; use of metric symbols mm^2, cm^2, m^2, km^2, ha; estimating	• finding the area of a figure on a grid by counting squares; area of a surface by tracing and counting squares; area as the amount of surface covered	• formulae: e.g., $A = lw$ $A = \frac{bh}{2}$
d) Volume; use of metric symbols mm^3, cm^3, m^3, km^3; estimating	• calculating area from linear measures; such as for a rectangle, square, and triangle • estimating, then calculating, the area • Pick's Theorem • using centimetre cubes packed into a variety of shells, then counting the number of cubes • volume as the amount of space occupied • volume of rectangular solids from linear measures and then calculation • estimating, then calculating volume	
e) Capacity; use of metric symbols mL, L; estimating	• using standard capacity containers to aid in determining the capacity of irregular containers • liquid displacement as an aid in determining volumes of irregular solids • estimating capacities • relationship between mass, capacity, and volume units	
f) Mass; use of metric symbols mg, g, kg, t; estimating	• mass by balancing using a set of graded metric masses; use of scales • estimating mass, then checking	• idea of conservation of mass

LEVEL 3 GRADE 9 MATHEMATICS (BASIC)

NOTES FOR TEACHERS

TRAND GEOMETRY

TOPICS	SUGGESTED DEVELOPMENT	RELATED TOPICS, CONCEPTS, SKILLS
SECTION 1 BASIC NOTIONS OF GEOMETRY		
) The language of geometry	• using geometric terms in their correct context • classifying angles as acute, right, obtuse, straight, reflex • classifying triangles as scalene, isosceles, right-angled, equilateral, acute-angled, obtuse-angled	• measurement • graphing
) Perpendicular and parallel lines	• classifying quadrilaterals as trapezoid, parallelogram, rectangle, square, rhombus, or kite • properties of parallel lines; alternate angles, corresponding angles, and supplementary angles formed by a transversal	
) Symmetry	• observing line, point, and turn symmetries of plane figures	
) Properties of plane figures	• triangle properties involving sides, angles, sides and angles, congruence • quadrilateral properties involving sides, angles, sides and angles • Pythagorean Theorem	• measurement • numerical applications
SECTION 2 CONSTRUCTIONS		
) Use of protractor	• measuring angles; constructing an angle of given measure; construction of triangles using a protractor, straight edge, and compass • determining angle properties of various geometric figures; angle sums; perpendicularity	
) Accurate constructions using a variety of instruments	• constructing congruent segments, congruent angles, perpendicular lines, perpendicular bisector, angle bisector, line parallel to a given line • constructing rectilinear figures; sufficiency conditions for a unique triangle, unique rectangle; constructing special cases of triangles and quadrilaterals • constructing a circle; locating the centre of a circle and of a circular arc; concentric circles • using a compass, straight edge, protractor, set square, tracing paper, paper folding, transparent mirror, parallel rule, etc. to construct many of the above figures	• data presentation
SECTION 3 TRANSFORMATIONS		
a) Slides, turns, and flips	• determining the image of a figure under a slide, turn, flip • using in context the terms translation, rotation, and reflection to describe the images obtained from slides, turns, and flips • properties of translations, rotations, reflections • accurate constructions of translation images, rotation images, reflection images • recognizing translation, rotation, and reflection images in the real world	• accurate constructions • measurement
b) Enlargements and reductions	• enlarging and reducing geometric figures using geoboards, dot paper, tiles, and/or grids • distinguishing the notions of shape and size; shape remains the same under enlargement or reduction • discovering properties of similar figures • determining the scale factor for an enlargement or a reduction; deciding on a suitable scale for enlarging or reducing • applying scale drawing and reading to real world situations, such as floor plans and maps	• accurate construction • measurement
c) Distortions	• using a squared grid and a distorted grid to draw a figure and its distorted image; changes in size, shape, area, etc. • examples of distortion images in the real world; e.g. images in non-flat shiny surfaces, funhouse mirrors, geoboard figures from elastics, stretches, etc.	

STRAND ALGEBRA

TOPICS	SUGGESTED DEVELOPMENT	RELATED TOPICS, CONCEPTS, SKILLS
SECTION 1 PROBLEM SOLVING		
a) Analysing and solving problems	• developing a strategy for solving problems; for example, — decide on focus: assessing whatever data is known and deciding what information you wish to find — interpret data: analysing the known information and its usefulness in developing the required information — execute and evaluate: using the given information to obtain results; deciding if the results satisfy the aim; a solution — extend the known data: interpreting available results and determining the necessary additional information: obtaining additional data: return to the first step and proceed as before	• model building
b) Aids to problem solving: understanding mathematical words,	• working with various mathematical instructions and interpreting them operationally: sum, difference, product, quotient, increased by, decreased by, fraction of, and so on	
diagrams,	• drawing a diagram whenever the problem can be clarified or analysed with one, such as in problems involving geometric figures and motion	
charts and tables,	• organizing information that is known into related sections of a chart, then determining possible strategies from the chart	
formulae,	• recognizing the nature of the problem and referring to its generalized solution as given by a formula • substituting into the formula • developing formulae from diagrams, and from patterns	
flow diagrams,	• using informal flow diagrams to organize the necessary steps required to solve multi-step problems	
calculators	• using a calculator to expedite computation, and to avoid error when extensive calculations are required • checking results with a calculator	
c) Equations: concept of variable,	• using a letter (variable) to represent the solution to a problem as an aid to understanding and organizing the known information	
forming equations,	• expressing information in a problem as an equation involving a single variable	
solving equations	• solving an equation by systematic trial using a calculator	

:AND ALGEBRA

TOPICS	SUGGESTED DEVELOPMENT	RELATED TOPICS, CONCEPTS, SKILLS
TION 2 INTRODUCTION TO COMPUTING		
Uses of a computer	• discussing in general terms the jobs a computer can do; such as calculating, memory, repeated operations	
	• recognizing the dependence the computer has on human input and instruction	
Working with a computer	• understanding that the essential operations of a computer are the four arithmetic operations together with comparison	
	• communicating with the computer: the need for a program; programming language; machine language	
	• discussing the general nature of a simple language such as Basic; interpreting simple problems in the language	
	• running a simple program if the equipment is available	
Binary System	• "on-off" nature of circuits requires use of binary system	
	• symbols of binary system; place value; simple conversions; operations in binary	
Uses of flow charting	• discussing the idea of a flow chart to organize a process; informal arrow charts; using symbols; informal flow charting of non-mathematical situations such as going home from school; flow charting of mathematical problems such as evaluating $3M^2$ when $M = 5$	
	• using flow charts to give instructions for a problem	
	• developing the flow chart for a problem as part of the solution to the problem	
TION 3 THE MATHEMATICS OF CHANCE		
Probability in simple two-outcome experiments	• developing an intuitive notion of likelihood, based on experience and data interpretation	• data presentation • computation
	• probability as a measure of likelihood; use of numbers from 0 to 1 inclusive, and percent, to represent probability	
	• probabilities associated with the toss of a coin, a selection from a sampling bottle with known composition, and so on	
	• studying situations which have a probability of 0, a probability of 1	
Probability in experiments with many possible outcomes	• exploring probabilities for experiments with many possible outcomes; such as, roll of a die, roll of two dice, sampling from bottles with known composition and with many outcomes	• notation: P(E) for the probability of event E
	• observing the probability of an event as given by "successful outcomes: total number of outcomes" (assuming each outcome is equally likely to occur)	
Sample space by listing	• analysing experiments where all possible outcomes can be listed easily and the probability immediately obtained by counting; e.g., given a list of 26 flavours of ice cream, find the probability of selecting a flavour containing nuts	• data presentation • counting
Sample space by a tree diagram	• analysing situations which are difficult to list without organizing; tree diagram as an organizing device to ensure all possible outcomes	
	• probabilities from tree diagrams; e.g., the probability of 3 boys in a 3 child family	
	• sample space defined as the set of all possible outcomes	

STRAND ALGEBRA

TOPICS	SUGGESTED DEVELOPMENT	RELATED TOPICS, CONCEPTS, SKILLS
SECTION 3 THE MATHEMATICS OF CHANCE (Cont.)		
e) Size of sample space by the Fundamental Counting Principle	• extending observations from tree diagrams to include the notion that for a large number of possible outcomes it is sufficient to count the possible and the successful; the Fundamental Counting Principle: e.g., find the probability of a licence plate in Ontario with two A's and two 3's, if all outcomes are possible.	• arithmetic of whole numbe • powers; evaluation of powe
f) Sampling for composition	• experimenting with a population where the composition is unknown; sample randomly, and replace; tabulate; use experimental observation as real composition	
g) Probability based on experiments	• extending experimental data to represent the composition of the total population gives a "ball park" measure of the composition of the population; cautions of overuse and misuse of this measure	• work with whole numbers, fractions and decimals • problem solving
h) Odds	• using the ratio of "successful outcomes: unsuccessful outcomes" as a measure of relative probability (or, more usually, risk); recognizing the larger the ratio, the smaller the risk • odds from probabilities; examining odds in a game of "overs and unders", roulette, and so on	

RAND GEOMETRY

TOPICS	SUGGESTED DEVELOPMENT	RELATED TOPICS, CONCEPTS, SKILLS
CTION 4 THREE-DIMENSIONAL GEOMETRY		
Solids, shells, and skeletons	• identifying real world objects as solids, shells, or skeletons • constructing shells for solids, skeletons for solids	• accurate constructions • measurement • vocabulary: vertex, edge, face, surface, point, line, plane, space, etc.
Polyhedra	• identifying polyhedra in the real world • decomposing shells of polyhedra to determine their nets • constructing regular polyhedra from their nets	• filling space
Pyramids and prisms	• distinguishing pyramids and prisms • constructing skeletons of pyramids and prisms from given nets; developing the nets	
Sketching 3-D objects	• observing 3-D objects from different perspectives; front, side, and top views in particular • sketching 3-D objects from different perspectives	• scale drawings

JGGESTED REFERENCES

ale Seymour et al, *Aftermath Series* (Setsco Educational Ltd.)
eve and Janis Marcys, *Mathimagination* (Setsco Educational Ltd.)
arold Jacobs, *Mathematics: A Human Endeavor* (W. H. Freeman and Co.)

APPENDIX 1

LEVEL 3 GRADE 9 MATHEMATICS (BASIC)

THEMES

This module was developed from ideas contributed by the following people at the brainstorming session held in March 1977, and convened by Claire Zeller.

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September 1977

LEVEL 3 GRADE 9 MATHEMATICS (BASIC)

APPENDIX 1

THEMES

CONTENTS

APPENDIX 1

LEVEL 3 GRADE 9 MATHEMATICS (BASIC)

INTRODUCTION

This resource module provides a number of *Themes* and *Subthemes* that could be used as a basis for developing many of the skills and concepts listed in the *Outline of Topics* for *Level 3 Grade 9 Mathematics (Basic).* Many students who will be taking this course have experienced frustration and failure in their attempts to learn the mathematics of earlier courses. The use of themes at this time may provide a means of awakening their interest in learning these skills and concepts in situations in which the need for the skills is established first.

The list of themes and subthemes is suggestive only, and by no means is complete. For example, themes related to sports, the music world, owning a car (see Appendix 2), and travel might be developed. Teachers are encouraged to develop some of their program through activities and learning experiences related to the themes suggested here or to themes of their own choice. The ideas and activities used should be appropriate for their students and should be effective for introducing and consolidating the skills under investigation at a particular time.

THEME 1 INCOME

1) Earnings of prominent people
2) Wages for part-time jobs, summer employment
3) Male-female wage differentials
4) Minimum wages, student wages
5) Commission; piecework; salary
6) Fringe benefits, payroll deductions
7) Pyramid selling
8) Franchise stores
9) Gimmick and creative selling; e.g., pet rock
10) Patent laws, copyright laws
11) Application for grants
12) Unemployment insurance
13) Student welfare
14) Income tax, baby bonus

THEME 2 NEWSPAPERS

1) How to read the newspaper, sections
2) How to find particular things, index
3) Measuring pictures; golden ratio
4) Interpreting graphs
5) Approximations
6) Large numbers
7) Word problems leading to a description of arithmetic operations
8) Problem solving
9) Distortions
10) Percentages from ads, sports
11) Frequency counts (of a letter, of a word)
12) Operation of division from ads and costing studies
13) Sifting data; extraneous data
14) Making assumptions; extraneous data
15) Realistic answers; estimation and approximation
16) Collecting data, prioritizing data
17) Displaying data, graphs
18) Graphical solutions
19) Interpolation and extrapolation
20) Lists, tables, combining tables
21) Reading tables
22) Non-computational uses of numbers (ordinal, etc.)
23) Income from newspaper routes

24) Mathematics of a paper route; maps; measurement; shortest route; best time to collect (statistics); Christmas for the newspaper person
25) Good news and bad news; what is good news? bad news?; ratio between good and bad
26) Ratio of spaces for advertising, pictures, sport, print, comic strips, etc.
27) Newspaper reflects the "real world"; math is in the newspaper, thus math is part of the "real world".

THEME 3 COUNTING PROBLEMS

1) Probability of winning a lottery
2) Phone numbers
3) Licence plates
4) "Telephone book" mathematics
5) Percentage return of a survey
6) Number of lock combinations
7) Tree diagrams
8) Taking samples
9) Impulse buying
10) Winning a lottery; what to do with the winnings
11) Making up a pool

THEME 4 SUPERMARKET MATH

1) Stacking space (e.g., with cubes, hexagonal boxes, prisms, etc.)
2) Inventory
3) "Fifo" inventory evaluation ("fifo" means "first in — first out" pricing)
4) Shapes
5) Volume
6) Packaging efficiency
7) Giving directions
8) "Deals" (20% more, "buy two, get one free")
9) Loss leaders
10) Unit price
11) Rain checks
12) Newspaper shopping
13) Inflation (C.P.I. Market Basket)
14) Part-time jobs in supermarkets
15) Conversions to metric (SI)
16) Comparative prices during conversion

THEME 5 TELEVISION AND MOVIES

1) Types of programs, frequency of occurrence, time of day
2) Ratings; polls; surveys; graphs
3) Canadian content regulations; types of programs
4) Viewing times; ratio of time for commercials to time for program, controls on advertising
5) Appearing on a talk show; fee; expenses; methods of income tax payment
6) Commercials: prime time costs; types; costs of making commercials
7) World wide telecasts; sponsor; types of programs; satellite transmission; time zones
8) Viewing habit surveys
9) Earnings of T.V. personalities
10) Cost of owning vs. leasing a T.V. set
11) Size of viewing screen (Pythagoras), diagonal measure, area
12) Cable T.V.: costs, advantages, converter
13) Making a T.V. program
14) Making a movie; gross: cost; frames per second; width of frames; length and running times
15) Scheduling shows; possibilities (tree diagrams)
16) Re-runs; low cost; non-prime time
17) Residual rights
18) Advertising: cost of T.V. advertisements; cost per viewer; the commercials industry
19) Pay T.V.; bargain?

20) Using T.V. and movie personalities in word problems; information from T.V. guide, T.V., and movie magazines
21) Give-away shows; value of prizes; taxes; strategies for winning
22) Sports telecasts; costs; technical techniques; instant statistics; sound pickup
23) Prime time vs. non-prime time; relative costs; return on investment
24) Home movies; equipment; techniques; zoom features; costs

THEME 6 SPECIAL DATES

1) Frequency of holidays; span between holidays
2) Calendar mathematics
3) Predicting dates; use of formula
4) Why there is a leap year; omission every 200 years
5) History of calendar; sense of time
6) Pay days: weekly, biweekly, etc.; advanced vs. delayed payments; money saved by delayed payment
7) Symbols: heart; shamrock, etc.; symmetry of symbols
8) Logos; metric and non-metric aspects; design; etc.
9) Budgeting for: Christmas, a holiday, "fly now-pay later", etc.
10) Geometric forms associated with holidays and seasons: snowman, leaf, poinsettia, etc.
11) Scheduling of holidays and days off
12) Age graphs
13) Cultural exchange; which cultures have which holidays and when
14) Dates: wars, number of people killed in wars, Remembrance Day
15) Timing devices; history of timing devices
16) Probability of the same birthday within the class

THEME 7 NUMBER PATTERNS

1) Geometric number patterns
2) Figurate number patterns
3) Inductive reasoning
4) Relations (how many ordered pairs can be formed); Cartesian products
5) Tiling; mosaics
6) Sequences of numbers
7) n^{th} term in simple sequences; generalizing arithmetic as an introduction to algebra
8) Fibonacci sequence; other sequences with recursive patterns
9) Simple patterns from +, −, x, and ÷ tables*
10) Rapid calculation tricks
11) Chain letters; tree diagrams
12) Magic squares; etc.
13) Patterns in tiling, number of tiles required to form regular polygons
14) Calculator activities and games

* See Stanley Bezuska; *Contemporary Motivated Mathematics;* S. J. Bezuska, M. J. Kenney; *Motivated Mathematics Series,* and *Activity Booklets;* Boston College Mathematics Institute, Chestnut Hill, Mass 02167

THEME 8 SHAPES IN THE WORLD AROUND US

1) Architectural designs
2) Man-made vs. natural shapes
3) Geometric patterns in nature
4) Curve fitting; tiling; stacking space
5) Curve stitching
6) 3-D models; building shells from nets; building skeletons from sticks and "joiners"; creative designs
7) Symmetry: in nature (tree's leaves, etc.); design of autos, furniture, etc.
8) Distortions
9) Escher Art
10) Art forms in public buildings
11) Bridges: structures, suspension bridges, supporting arches
12) Logos and crests: commercial logos
13) Traffic signs; international road signs

14) Highway designs; clover leaf; centrifugal force; centre of gravity
15) Interior designing
16) Curved space
17) Surveying
18) Bricklaying designs
19) Packaging of commercial products
20) Special buildings: Toronto City Hall, CN Tower, etc.
21) Rigid structures and applications; non-rigid structures and applications
22) Mazes, maze games
23) Geometric puzzles; e.g., Looney Loop, wire puzzles

Books: *Pattern in Nature;* Peter Stevens
Language of Math; Frank Land
Search for Pattern; W.W. Sawyer
Beyond Solitaire; Sid Sackson

THEME 9 BANKING

1) Charter banks, trust companies, credit unions — similarities, differences
2) Types of accounts
3) Opening an account
4) Deposit forms
5) Cheques, personal record-keeping
6) Bank account statement; reconciliation
7) Interest: rate, percent; simple interest, compound interest
8) Loans; interest days; days of grace on loans
9) Service charges
10) Borrowing; interest charges; hidden costs; legal controls
11) The "360 day" year
12) "Number of days between" calculations
13) Daily interest tables
14) Monthly interest tables
15) Bank cards (credit, identification); charges for late payment
16) Careers in banking
17) Sex role stereotyping
18) Mortgages
19) Credit: types of credit; length of credit periods; borrowing on one's income tax
20) Money drafts
21) Foreign exchange

THEME 10 STAMPS

1) Shape, area, perimeter of stamps
2) Value of stamps
3) Trading stamps
4) Discounting coupons, discount money
5) Illustrations of mathematics on stamps
6) Postage rates: Canada, U.S.A., foreign
7) Mathematics as illustrated in nature on stamps
8) History of stamps
9) Effect of currency changes on stamps; conversions of stamps of foreign countries to Canadian value
10) Effects of conversion to metric system on stamps
11) Appreciation of value of stamp collections; investment possibilities; trading
12) Costs of collecting stamps
13) Special stamps: new issues; limited printing; special events; etc.

THEME 11 CALCULATORS

1) Price of calculators; comparative shopping, discounts
2) Cost of operation with batteries, adaptor, re-chargeable batteries, liquid quartz system

3) Sophistication of calculators; special features; functions
4) Relative life of different models, of different qualities of batteries
5) Machine algebra
6) Hardware: circuits, logic circuits
7) Repeating decimals
8) Percent pay; how it works; what it does
9) Flow charting
10) Using the calculator for complex calculations; using the memory
11) Estimating: testing the estimate
12) Powers of ten
13) Big numbers, scientific notation; small numbers, scientific notation
14) Complicated numbers
15) Automatic constants; using them
16) Teaching exponents
17) Simulating a computer
18) Reconciliation of bank accounts
19) Developing algorithms
20) Decimals
21) Programmable calculators
22) Remediation activities on programmed calculators; "Quiz Kid"*, "Little Professor"**
23) Calculator games; activities
24) Inverse operations, input – output

* Natural Semiconductor Products (Novus)

** Texas Instruments Inc.

THEME 12 VEHICLES AND ACTIVITIES

(Snowmobiles, cars, motorcycles, vans, etc.)

1) Initial cost of vehicle
2) Cost of licence plates
3) Finance charges: bank loans, finance companies, credit unions, etc.
4) Fuel costs; speed vs. economy
5) Insurance
6) Maintenance costs: expected costs, unexpected costs
7) Depreciation
8) Perimeter, area, volume
9) Distance, rate, time
10) Appreciation of cars (antique cars as an investment)
11) Personalizing vans
12) Time spent in/on vehicle
13) C.B. radio: cost, benefits, effectiveness
14) Racing
15) "Pencil point 500 game"
16) Leasing vs. owning
17) Taxis vs. owning
18) Public transit vs. owning
19) Pollution
20) Drinking drivers; breathalizer tests
21) Fines: cost for different offences; costs of going to court; point system
22) Design of cars
23) Road maps
24) Scale drawings of cars, parts of cars; car manuals; repair books
25) Rallying
26) Brakes; disc brakes vs. drum brakes; stopping distance vs. speed vs. mass
27) Snow tires; effectiveness vs. cost
28) Undercoating; effectiveness vs. cost
29) Trailers: licence, hitches, running lights, traffic management, effect on fuel consumption, insurance, maintenance
30) Travel costs; camping vs. motels

THEME 13 BUYING INTELLIGENTLY

1) Unit cost: price per oz., per gram, etc.; economy size?
2) Budget
3) Timing of sales
4) Credit; cash; cheques
5) Sales; seasonal buying
6) Discounts; percent off
7) Leasing or renting; e.g., T.V., stereo, car, etc.
8) Using newspaper information wisely
9) Comparative buying from store to store, product by product, within the same store
10) Shopping calculators
11) Estimating
12) New checkout systems; coded labels
13) Catalogue buying
14) Total cost of an article; purchase price, service costs, transportation cost, service contracts, insurance
15) Billing procedures
16) Profit margins
17) Bartering
18) Auctions; dutch clock
19) Export — import; exchange rate; balance of payments
20) Sales tax, federal taxes
21) Cost of freezer vs. savings on food costs
22) Credit rating; advantage of good credit rating

THEME 14 MATH GAMES

STRATEGY GAMES	SKILL DEVELOPMENT GAMES
1) Battleship	1) Bingo
2) Backgammon	2) Battleship
3) Mastermind	3) Monopoly
4) NIM	4) Stockticker
5) Tangrams	5) Krypto
6) 3-D Tic Tac Toe	6) Tuff
7) Stockticker	7) Card games
8) Yahtzee	8) Dice games
9) Mancola	9) Calculator games
10) Pythagoras	
11) Chess	
12) Checkers	

Books: *Beyond Tic Tac Toe;* Sid Sackson
Beyond Solitaire; Sid Sackson

Suppliers: Edu-media Ltd.; Kitchener, Ontario N2G 4H1
J. Weston Walch, Publisher; Portland, Maine 04104
Setsco Educational Ltd.; Scarborough, Ontario M1S 3B4
Western Educational Activities Ltd.; Edmonton, Alberta T5H 2L4

THEME 15 MAPS AND GRAPHS*

1) How to lie with statistics
2) Distortions
3) How to interpret maps
4) Everyday graphs (particularly step graphs)
5) How to make maps
6) Giving directions
7) Scale diagrams
8) Interpretation of scale

9) Measurement
10) Making maps of local areas
11) Dilatations
12) Floorplan of school
13) Coding; e.g., how your school is coded
14) Blueprints
15) Floorplans of a house
16) Distances; best ways to travel

* This theme provides a good review, because it relates to many other topics.

SUGGESTED REFERENCES

Dale Seymour et al, *Aftermath Series* (Setsco Educational Ltd.)
Steve and Janis Marcys, *Mathimagination* (Setsco Educational Ltd.)
Harold Jacobs, *Mathematics: A Human Endeavor* (W. H. Freeman and Co.)

LEVEL 3 GRADE 9 MATHEMATICS (BASIC)

MATHEMATICS AND THE CAR

a sample development of materials related to

Theme 12: Vehicles and Activities

NOTE: *This module is based on materials that were created for students at Westwood S.S. in 1975. At that time, Mr. Teeter was the principal, Mr. Zeller the mathematics head, Mr. Nowak the assistant mathematics head, and Ms. Vigoda a popular mathematics teacher. The facts concerning their cars were authentic. Rock Starr is symbolic of the student dream. The material in its present form was prepared by the Mathematics Department of the school. Its success was based on the students' interest in cars, and also their interest in the lives of the personalities who owned them. It is suggested that these materials be revised and printed at the individual school level using the same type of "local colour".*

September 1977

MATHEMATICS AND THE CAR

Every young person longs to drive a car and then, of course, he/she longs to own one. Students should consider the advisability of owning a car, and study the costs of buying, repairing, insuring, and driving it.

A. BUYING A CAR

A person may pay cash for a car, or he may pay part of the cost in cash and borrow the remainder. If a purchaser wishes to borrow money to pay for the car, he may borrow the money from: a friend, a relative, a bank, or a finance company. Usually the loan is paid back in monthly installments (payments) which include both interest and payment of part of the original loan (the principal).

1. Mr. Nowak bought a one-year-old Honda Civic for his girlfriend Zelda Puggenowski. The price was a real steal, only $1995.00. Find the total cost of the car to Mr. Nowak.

PRICE .....

SALES TAX (at 7%) .....

TOTAL PRICE .....

2. Ms. Vigoda recently purchased her Vega "Hatchback" brand spankin' new. She paid $4150.00 for the car, but made a down payment of $1000.00 and borrowed the remainder from her T-D Bank. She is repaying the loan at the rate of $135.00 per month for 30 months, plus a final payment of $65.55. Find the total cost to Ms. Vigoda of this "golden jet", and calculate the cost of borrowing the money.

PRICE $4150.00

SALES TAX (at 7%) .....

TOTAL PRICE (for cash) .....

VALUE OF PAYMENTS _____ x _____ = _____

LAST PAYMENT _____

TOTAL AMOUNT TO BE PAID BY PAYMENTS _____

DOWN PAYMENT _____

TOTAL AMOUNT PAID _____

COST OF BORROWING _____ x _____ = _____

3. Ms. Vigoda could have financed the car in question # 2 through the car dealer. Ms. Vigoda was told that if she paid $800 down, she could pay $105.00 per month for 36 months and then a final payment of $67.50. Find the total cost of the car and the finance charges under this method of payment.

4. An ad in the *Globe and Mail* says that a $2495.00 car can be bought for $800.00 down and 30 monthly payments of $63.75. What are the finance charges? (Ignore the tax).

5. Mr. Teeter bought a car in Quebec where the sales tax was 8%. The Chrysler Cordoba cost $8350.00. Compare the total cost of the car under the following plans:

a) Cash payment (including tax)

b) $2000.00 down and the balance borrowed from the bank, repayable at $205 per month for 3 years.

c) $2500.00 down and the balance financed through the dealer and repayable at $152.00 a month for 45 months.

6. When Rock Starr bought his new Continental, the dealer allowed him $6400.00 for his 1972 Mercedes-Benz. The cost of the Continental was $14 676.00. Compare what Rock would have to pay under the following payment plans:

a) Cash payment including 7% sales tax?

b) A bank loan repayable at $400.00 per month for 2 years?

c) Financing through the dealer at $376.00 for 30 months?

B. DEPRECIATION

A car costing $3000 might be valued at $2000 after one year of driving. At the end of two years it might be worth $1400 and at the end of three years worth only $900. This decrease in value as a result of wear and tear and model changes is called depreciation.

COST OF CAR	1st Year	2nd Year	3rd Year	4th Year	ADDITIONAL YEARS
$0 – $3000	27%	20%	15%	12%	10%
$3000 – $5000	30%	27%	20%	15%	12%
$5000 – $8000	32%	28%	25%	18%	15%
over $8000	30%	30%	22%	20%	18%

1. This table shows the rate at which a car of a particular value depreciates.

a) Rate of depreciation on a $4700 car in 2nd year is _ _ _ _ _ _ _ _ _ %

b) Rate of depreciation on a $9500 car in 8th year is _ _ _ _ _ _ _ _ _ %

c) Rate of depreciation on a $6500 car in 1st year is _ _ _ _ _ _ _ _ _ %

d) Rate of depreciation on a $2995 car in 3rd year is _ _ _ _ _ _ _ _ _ %

e) Rate of depreciation on a $1200 car in 5th year is _ _ _ _ _ _ _ _ _ %

f) Find the first year depreciation on Zeller's 1976 Trans Am costing $8100.

g) What is the Trans Am worth at the end of the first year?

$8100 - _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ = _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _

(amount of depreciation)

NOTE: In each year, the amount of depreciation is always a percent of the depreciated value for the previous year.

h) Find the second year depreciation on the Trans Am:

_ _ _ _ _ _ _ _ _ _ _ _ _ x _ _ _ _ _ _ _ _ _ _ _ _ _ = _ _ _ _ _ _ _ _ _ _

(depreciated value; i.e., answer in g))

(2nd year depreciation rate)

i) What is the "super bird" worth after 2 years?

_ _ _ _ _ _ _ _ _ _ _ _ _ - _ _ _ _ _ _ _ _ _ _ _ _ _ = _ _ _ _ _ _ _ _ _ _

j) Calculate the value of the "FIREBIRD" after:

- 3 years _ _ _ _ _ _ _ _ _ _ _ _
- 4 years _ _ _ _ _ _ _ _ _ _ _ _
- 5 years _ _ _ _ _ _ _ _ _ _ _ _
- 6 years _ _ _ _ _ _ _ _ _ _ _ _

2. Using the method described in f), g), h), i), j), and the table above, complete the following table:

AGE	HONDA CIVIC $1995 (1 year old)	VEGA "HATCHBACK" $4150 (new)	CONTINENTAL $14 676 (new)
1st Year	Dep: Value:	Dep: Value:	Dep: Value:
2nd Year	Dep: Value:	Dep: Value:	Dep: Value:
3rd Year	Dep: Value:	Dep: Value:	Dep: Value:

Dep: represents the depreciation

3. Remember Mr. Teeter's Chrysler Cordoba? Well, it's getting older; in fact, it's 6 years old. Calculate how much he should expect to be allowed on a trade. (Cost: $8350 new)

		VALUE
COST:		$8350
Less (1st year)	_____ =	_____
Less (2nd year)	_____ =	_____
Less (3rd year)	_____ =	_____
Less (4th year)	_____ =	_____
Less (5th year)	_____ =	_____
Less (6th year)	_____ =	_____

(the probable trade-in value)

What percentage is the total depreciation of the original value of the car?

$$\frac{\text{cost} - \text{"trade-in"}}{\text{cost}} \times 100\% = \_\_\_\_\_\%$$

C. OPERATING EXPENSES (TOO MUCH!)

1. Licence plates are sold according to the number of cylinders in the engine:

4 cylinders	$30.00
6 cylinders	$40.00
8 cylinders (or more)	$60.00

NOTE: These are the proposed rates for 1978.

Find the total cost of licence plates for all the cars studied so far:

Honda Civic	4 cylinders
Vega "Hatchback"	6 cylinders
Chrysler Cordoba	8 cylinders
Continental	8 cylinders
Trans Am	8 cylinders

2. Insurance is usually sold on a half-yearly or a yearly basis; sometimes monthly installments are permitted on these rates.

a) Mr. Zeller pays $32.00 per month for insurance on his Trans Am. How much is that half-yearly? How much yearly?

b) Ms. Vigoda pays $24.00 per month for her Vega "Hatchback". How much does she pay per year?

c) Mr. Teeter pays $336.00 per year. How much does he pay per month?

d) If Rock Starr pays $426.00 for a half-year, what monthly payment would be needed?

3. Service: If oil costs $1.10 per litre, a grease job is $3.00, and cars are greased and have their oil changed every 5000 km, complete the table below.

CAR	NUMBER OF KILOMETERS PER YEAR	OIL CAPACITY	OIL COST PER YEAR	GREASE COST PER YEAR	TOTAL COST
Honda Civic	30 000	3 L			
Vega Hatchback	30 000	3 L			
Chrysler	15 000	4 L			
Trans Am	45 000	4 L			
Continental	30 000	5 L			

4. From the information above, compute the monthly cost of normal service, insurance, and licence for:

a) Ms. Vigoda's Vega

b) Mr. Zeller's Trans Am

c) Mr. Teeter's Chrysler Cordoba

d) Rock's Continental

D. FUEL CONSUMPTION

Perhaps the most important aspect of a car in these days of gas "shortage" is its fuel consumption; i.e., the number of kilometres the car travels on a litre of gasoline. This is comparable to the number of miles per gallon in the British system.

$$\text{Fuel consumption (km/L)} = \frac{\text{Distance travelled (km)}}{\text{Volume of fuel used (L)}}$$

or as a formula, $C = \frac{d}{V}$ using the units indicated above.

The formula may be written in three ways.

a) $C = \frac{d}{V}$ b) $d = CV$ c) $V = \frac{d}{C}$

1. Assume a car gets 8 km/L.

- How far can it go on 35 L?

C = _____ V = _____ d = _____

Which formula a), b), or c) did you use?

- How many litres are needed to go 300 km?

Which formula a), b), or c) did you use?

Suppose after a tune-up the car goes 1000 km on 102 L.

- What is the new fuel consumption?

d = _____ V = _____ C = _____

Which formula a), b), or c) did you use?

2. Complete the following table using the formulas $C = \frac{d}{V}$

$d = CV$, and $V = \frac{d}{C}$. Complete the answers to one decimal place.

	d DISTANCE TRAVELLED (km)	V VOLUME OF FUEL USED (L)	C FUEL CONSUMPTION (km/L)
TRIP 1	75	15.0	5.0
TRIP 2	150	18.0	——
TRIP 3	300	———	12.0
TRIP 4	——	12.0	4.8
TRIP 5	——	9.0	13.5
TRIP 6	275	45.0	——
TRIP 7	330	———	4.2
TRIP 8	900	73.6	——
TRIP 9	——	36.4	11.7
TRIP 10	510	———	7.3
TRIP 11	80	13.6	——
TRIP 12	——	7.5	4.9
TRIP 13	225	19.8	——
TRIP 14	——	6.5	14.7
TRIP 15	821	130.0	——

3. Mr. Zeller's Trans Am gets a poor 3.1 km to the litre in city driving, and an amazing 4.8 km to the litre on the highway when he drives less than 100 km per hour (no burying the needle). During the week he does 440 km of highway driving and 192 km of city driving.

- How many litres does he use in a week?

- What does it cost for the week's gas, if he uses Jet "Lead Foot" at an average cost of 25.9¢ /L?

4. Mr. Teeter's Chrysler Cordoba gets an average of 4.2 km/L for normal driving, and 5.6 km/L for highway driving. In 1975 he drove a total of 22 400 km, 60% of which were normal driving and 40% of which were highway driving. How much gas did he use in 1975, and how much did it cost if the average price per litre was 26.9¢?

METHOD:

Number of kilometres of normal driving _____ % of _____ = _____

Number of kilometres of highway driving_____ % of _____ = _____

Number of litres for normal driving $\frac{\quad}{4.2}$ = _____

Number of litres for highway driving $\frac{\quad}{5.6}$ = _____

TOTAL COST_____ x $0.269 = _____

5. Zelda's Honda Civic is something else! She gets 14.1 km/L in city driving and 16.4 km/L on trips. In the last month she has gone 3000 km, of which 70% has been highway driving.

- Calculate how many litres of gas she has used .

- Calculate the total cost, if the average cost was $0.249/L.

6. Ms. Vigoda's Vega "Hatchback" gets 12.7 km/L when she drives 60 km/h, and exactly 11.9 km/L when she drives 100 km/h ("on the wood"). It is 540 km from Westwood to Montreal.

i) How much gas would it take to drive to Montreal at 60 km/h?

ii) How much gas would it take at 100 km/h?

iii) How much money would she save at the slower speed if she gets "cut-rate" at $0.199/L?

iv) How much time does she save at the faster speed?

$$\text{Time (h)} = \frac{\text{Distance travelled (km)}}{\text{Speed (km/h)}}$$

or as a formula, $t = \frac{d}{v}$ using the units indicated above

Write formulas for speed, distance, and time using v, d, and t.

i) d = _____ x _____

ii) v = _____

iii) t = _____

7. For Rock Starr's Continental the gas shortage counts. He gets 2.8 km/L in the city, 4.3 km/L on the highway under 80 km/h and a "cool" 3.7 km/L over 80 km/h. Rock goes on a 3200 km trip, 25% in the city, 65% on the highway under 80 km/h and the balance over 80 km/h. How much gas does he use and how much does it cost, if he uses Sunoco (WE KNOW CARS) 260 at 27.9¢/L?

METHOD:

Distance in the city = _____ x _____ km = _____ km

Distance under 80 km/h = _____ x 3200 km = _____ km

Distance over 80 km/h = 3200 km - _____ = _____ km

Volume of gas in the city = $\frac{\quad}{2.8}$ L = _____ L

Volume of gas under 80 km/h = $\frac{\quad}{4.3}$ L = _____ L

Volume of gas over 80 km/h = $\frac{\quad}{3.7}$ L = _____ L

TOTAL AMOUNT OF GAS USED = _____ L

TOTAL COST _____ x _____ = _____

SUGGESTED REFERENCES

Dale Seymour et al, *Aftermath Series* (Setsco Educational Ltd.)
Steve and Janis Marcys, *Mathimagination* (Setsco Educational Ltd.)
Harold Jacobs, *Mathematics: A Human Endeavor* (W. H. Freeman and Co.)

LEVEL 3 GRADE 10 MATHEMATICS (BASIC)

NOTES FOR TEACHERS

THEMES

AND

REFERENCES FOR THEIR DEVELOPMENT

NOTE: *This is a DRAFT COPY for validation purposes and for optional implementation during the school years 1977 – 1978 and 1978 – 1979. Reactions should be forwarded to the Intermediate Mathematics Coordinator, Ministry of Education, Curriculum Branch, Mowat Block, Queen's Park, Toronto M7A 1L2.*

September 1977

LEVEL 3 GRADE 10 MATHEMATICS (BASIC)

THEMES AND REFERENCES FOR THEIR DEVELOPMENT

CONTENTS

DEVELOPING A PROGRAM LOCALLY

The *Level 3, Grade 10 Mathematics (Basic)* program is designed to further reinforce the skills and concepts given in the *"Outlines of Topics"* for *Level 3, Grade 9 Mathematics (Basic).* This should be done by emphasizing real life applications that are relevant to the students now and will be useful to them in later life.

The nine *Themes and Subthemes* provided in these notes are intended to assist teachers in planning programs that have a practical emphasis. Even though the skills and concepts for Grade 9 form the roots of this program, the vehicles for showing their importance should be different than those used in the Grade 9 course. These themes contain an abundance of implicit mathematical knowledge and requisite skills. It is not expected that all these themes, or their subthemes, will be developed with the students; rather the program should reflect the interests of the students and should be rich with local situations and experiences.

Under normal circumstances it is anticipated that an interesting and valuable course may be developed around four to six of these themes in one academic year. This of course is not binding, and teachers are encouraged to experiment with different combinations of appropriate ideas that will motivate the students of particular classes. In addition, teachers might explore local themes of their own creation, should they seem more suitable. For example, a unit on the CN Tower could be fruitful for students in Metro Toronto, where many of the students may have visited it or where a field trip to the tower could provide motivation and background for further study. On the other hand, a unit on commercial hunting and fishing might be more compelling for students in northern regions.

To indicate the sort of treatment that might be followed, the subthemes have been keyed to a bibliography of twelve reference books and two kits. This bibliography is deliberately brief; many other references within the experiences of an individual teacher may be used to augment this list. Local committees might develop student materials which may be shared with other teachers. Appendix 1 provides a partial bibliography of materials that are available from financial institutions. In addition, other resource modules, which have been developed by classroom teachers, are available from the Ministry of Education (Appendix 2, Level 3, Grade 9 Mathematics, Theme: *Mathematics and the Car;* Appendix 2, Level 3, Grade 10 Mathematics, Theme: *You and The Music World).*

The references in the above paragraph are not to be considered as policy, nor are they encyclopaedic.

1. THEME 1 MATH AT WORK AND AT HOME

	SUBTHEMES	REFERENCES*
a)	Pre-employment tests	A local test if one is available
b)	Mathematics at work	A, B, D, E, F, G, H, J, K, L
c)	Mathematics in inventory	A, L
d)	Gratuities; expected and real	H, L
e)	Commuting to work	A, L
f)	Fixed costs at home: rent, taxes, mortgages, etc.	A, K
g)	Variable costs at home: hydro, telephone, food, etc.	A, B, E, K telephone book
h)	Home improvements: repair people, do-it-yourself	telephone book A, B
i)	Entertainment costs: • at home — facilities, food, drinks, etc. • a "night out" — transportation, baby-sitting admission, cover charge, minimum, gratuities, taxes, etc.	catalogues, newspapers partying (see L)

2. THEME 2 YOUR EARNINGS AND YOUR BUDGET

	SUBTHEMES	REFERENCES
a)	Methods of payment: hourly, salary commission	A, B, C, E, G, H, K, L, M, N
b)	Deductions from pay: C. P. P., U. I., Income Tax, etc.	A, B, E, F, G, H, K, M, N
c)	Constructing a budget: fixed expenses, flexible expenses, savings	B, E, G, H, J, K, M, N
d)	Intelligent buying	A, B, J, K, L, M, N
e)	Computing your net worth; depreciation	M, N

* See list of references on page 7.

3. THEME 3 YOU AND FINANCIAL SERVICES*

	SUBTHEMES	REFERENCES
a)	Types of savings and loan institutions	B, M, N
b)	Types of accounts	B, E, G, J, K, M, N
c)	Opening an account; bank forms	A, B, J, K, M, N
d)	Reconciling your account	A, B, C, E, G, J, K, M, N
e)	Interest	A, B, C, D, E, G, H, I, J, K, M, N
f)	Making a personal loan	A, B, D, E, G, H, K, M, N
g)	Mortgages	B, E, K, M, N
h)	Credit cards and ratings	B, E, K, M, N
i)	Other services	B, M, N

* See Appendix 1, *A Partial Bibliography on Financial Institutions,* for further resources.

4. THEME 4 YOU AND TAXES

	SUBTHEMES	REFERENCES
a)	Sales Taxes: provincial differences; state differences; unconcealed and concealed nature of tax; non-taxable items	B, C, D, E, J, L, M, N
b)	Provincial Taxes: income tax (personal, corporate); product specialty taxes and licences; succession duties	B, E, J, K, M, N
c)	Municipal Taxes: components; concept of assessment; the mill	B, E, K, M, N
d)	Federal Taxes: excise taxes, customs and duties; income taxes (personal, corporate); capital gains taxes; and sales taxes	A, B, E, G, H, J, K, M, N *Teaching Taxes* (Dep't of National Revenue) *The Sloane Affair* (National Film Board)

5. THEME 5 YOU AND TRAVELLING – PUBLIC AND PRIVATE

	SUBTHEMES	REFERENCES
a)	Walking and hiking; giving directions; orienteering	C, H
b)	Riding a bicycle; type, initial cost, maintenance; mechanisms, gears, ratio	D, M
c)	Short trips on public transit (bus, taxi, subway)	A, D, E, G, M
d)	Automobile; ownership, expenses	A, B, D, E, F, G, H, I, L, M, N
e)	Using maps and itinerary planning	A, C, D, F, H, I, J, L
f)	Planning a vacation: • choosing a destination • comparing the travel costs (car, bus, air) • accommodation expenses • living expenses (meals, recreation, tips) • using a travel agent • budgeting vs. "fly now and pay later" • travel packages	A, B, E, J, M, N

6. THEME 6 YOU AND INSURANCE

	SUBTHEMES	REFERENCES
a)	Insuring your car	B, E, G, L, M, N
	• factors affecting the rate — territorial, characteristics of the insured and his vehicle	
	• types — liability, medical, collision, comprehensive	
	• legal limits	
	• no-fault insurance	
b)	Insuring your life	A, B, E, G, M, N
	• straight life — mortality table	
	• limited payment life	
	• life to 60, or life to 65	
	• term insurance (1, 5, 10 years; to 60, to 65)	
	• participating and non-participating policies	
	• endowments	
	• pension plans	
	• optional features — double indemnity, waiver of premium, guaranteed insurability, etc.	
	• cash values and borrowing	
	• family plans	
c)	Insuring your possessions (fire, theft, etc.)	A, B, E, M, N
	• home owners policies	
	• contents of apartments	
	• valuable possessions (diamonds, fur coat, etc.)	
	• boat	
d)	Health Insurance	E, M, N
	• O.H.I.P.	
	• extended health care	
	• dental plans	
	• Blue Cross	

7. THEME 7 MATHEMATICS IN SPORTS

	SUBTHEMES	REFERENCES
a)	Scoring sporting events	A, D, H, L
b)	Professional sports; nature of income from different sports	
c)	Rating the players, teams, participants	A, D, G, H, I, L
d)	Strategies	A
e)	Analysing the standards	C, D
f)	Schedules and playoffs	
g)	Geometry of playing surfaces	A, C, D, J, L
h)	Handicapping, probabilities, predicting the outcome, wagering, odds	F, G, L
i)	Records, Olympics, titles	D, H
j)	Simulating sports events: Pencil Point 500, Boof-Tal, Golf, T.V. screen electronic games, etc.	*Car and Driver,* August 1973 *Mathaction 6* (Copp Clark)
k)	Cost of equipment, and facilities	H

8. THEME 8 SETTING UP A PLACE OF ONE'S OWN*

SUBTHEMES

a) Room and board arrangements

b) Renting a room
- furnishing the room
- limited meal preparation (share, or hot plate)

c) Renting an apartment
- size, location, facilities, lease contract
- cost of convenience vs. commuting costs
- furnishing a complete apartment
- your food dollar
- insurance on contents; liability
- telephone, T.V. cable, utilities, and other monthly charges

d) Purchasing a condominium
- highrise apartment, new or resale
- townhouse or row house, new or resale
- income requirements, mortgages, common elements expenses, down payment
- legal advice and costs; deed; search of title
- insurance
- furnishing
- food budget
- insurance on contents; liability
- telephone, T.V. cable, common expenses, and other monthly charges

e) Purchasing a home
- size, location, lot, facilities, maintenance requirements (annually and long term)
- see d) for remaining considerations

* This theme should be developed with a local flavour that reflects local availability, costs, and preferences. Accordingly, references should be acquired from the local community.

9. THEME 9 YOU AND THE MUSIC WORLD*

SUBTHEMES

a) Cost of lessons (private, group)

b) Cost of equipment (instruments, music, records) buy or rent

c) Solo or as a group

d) Getting exposure; initiation fee and union dues

e) Role of an agent

f) Legitimate expenses deductible from earnings:
- travel
- clothes
- accommodation
- practice studio
- recording costs
- depreciation
- public relations
- union dues, telephone

g) Full or part time; income and income tax differences

* Only reference H touches this theme in even a general way. See the resource module "You and the Music World", soon to be available from the Ministry of Education.

h) Contracts:
- scale
- flat rates over scale
- flat rate plus percentage of gate
- percentage of gate

i) Promoting a concert or dance

j) Determining the "Hit Parade", "Top 40", etc.

k) Royalties and copyrights on records and commercial play, residuals

BIBLIOGRAPHY

BOOKS (REFERENCES)

A. Bolster, Woodburn: *Mathematics in Life;* Scott, Foresman
B. Conchie: *Business and Consumer Mathematics;* Addison-Wesley
C. Ebos, Tuck: *Math Is/3;* Nelson
D. Henderson, Attridge: *Starting Points in Mathematics 9;* Ginn
E. Kravitz, Brant: *Consumer Related Mathematics;* Holt, Rinehart and Winston
F. Lind, Caruso, et al: *Intuitive Mathematics;* Walmath Books
G. Littman: *Practical Problems in Mathematics for Consumers;* Delmar
H. Shaw, Wheatley, et al: *General Mathematics 1;* Houghton Mifflin
I. Sobel, Maletsky: *Essentials of Mathematics 2;* Ginn
J. Sobel, Maletsky: *Essentials of Mathematics with Consumer Applications;* Ginn
K. Tobin et al: *Mathematics For Today (Level Red);* Oxford Book Co.
L. Tobin et al: *Mathematics For Today (Level Blue);* Oxford Book Co.

KITS (REFERENCES)

M. *Money Management Library;* Household Finance Corporation
N. *Your Money Matters;* Royal Bank of Canada

SUGGESTED REFERENCES

Dale Seymour et al, *Aftermath Series* (Setsco Educational Ltd.)
Steve and Janis Marcys, *Mathimagination* (Setsco Educational Ltd.)
Harold Jacobs, *Mathematics: A Human Endeavor* (W. H. Freeman and Co.)

APPENDIX 1

LEVEL 3 GRADE 10 MATHEMATICS (BASIC)

A PARTIAL BIBLIOGRAPHY ON FINANCIAL INSTITUTIONS

The specific resources listed in this bibliography are samples only. Similar resources may be obtained from financial institutions in the local community.

NOTE: *This is a DRAFT COPY for validation purposes and for optional implementation during the school years 1977 – 1978 and 1978 – 1979. Reactions should be forwarded to the Intermediate Mathematics Coordinator, Ministry of Education, Curriculum Branch, Mowat Block, Queen's Park, Toronto M7A 1L2.*

September 1977

APPENDIX 1

LEVEL 3 GRADE 10 MATHEMATICS (BASIC)

	CONTACTS FOR MATERIALS, JUNE, 1977	PHONE	CODE
1.	Canadian Bankers Association P.O. Box 282 Toronto Dominion Centre Toronto, Ontario M5K 1K2	416 362 6092	CBA
2.	Bank of Canada 250 University Ave., Toronto, Ontario M5H 3E5	416 368 6092	BC
3.	Bank of Montreal First Canadian Place, Toronto, Ontario M5X 1A3	416 867 5050	BM
4.	Canadian Imperial Bank of Commerce Commerce Court Toronto, Ontario M5L 1G9	416 862 2211	CIBC
5.	Canada Trust 110 Yonge St., Toronto, Ontario M5C 1T4	416 362 6161	CT
6.	The Permanent 320 Bay St., Toronto, Ontario M5H 2P6	416 361 8000	P
7.	Provincial Bank of Canada 145 King St. W., Toronto, Ontario M5H 1J8	416 360 8400	PBC
8.	Royal Bank Royal Bank Plaza, Toronto, Ontario M5J 2J5	416 865 2603	RB
9.	Scotia Bank 44 King St. W., Toronto, Ontario M5H 1E2	416 866 6161 Ex. 6695	SB
10.	Toronto Dominion Bank Toronto Dominion Bank Tower, Toronto, Ontario M5K 1A2	416 866 8222	TD
11.	Ontario Credit Union League 4 Credit Union Dr., Toronto, Ontario M4A 2A1	416 759 4711	OCUL
12.	Money Management Institute Household Finance Corporation of Canada 85 Bloor St. E., Toronto, Ontario M4W 1B4	416 960 0665	MMI
13.	Trust Companies Association of Canada 11 Adelaide St. W., Ste. 400, Toronto, Ontario M5H 1L9	416 364 1207	TCAC
14.	College of Family and Consumer Studies, University of Guelph, Guelph, Ontario N1G 2W1		CFCS

TEACHERS' BIBLIOGRAPHY

1.	HISTORY AND CHARACTERISTICS OF BANKING INDUSTRY	REFERENCE
.01	Decennial Revision of Canada's Banking Acts	CBA
.02	Bill C-222	"
.03	The Bank Act	"
.04	Banking Statistics	"
.05	Consumer Credit and the Chartered Banks	"
.06	Education and the Banker	"
.07	Chartered Banking: The Informative Industry	"
.08	Banking in the 60's – A Statistical Review	"

.09	Credit on the Farm	CBA
.10	Finance Committee Highlights: The Banks' Presentation in Ottawa	"
.11	A New Payment Mechanism in the Making	"
.12	Tax White Paper: The Banks' View	"
.13	Bank Stock: Home Owned and Widely Held	"
.14	Homes for Canadians: Role of the Banks	"
.15	Growth, Service, and Vicissitudes	"
.16	Changing Patterns of Consumer Credit: Part I	"
.17	Changing Patterns of Consumer Credit: Part II	"
.18	President's Speech, 1972	"
.19	Canadian and U.S. Banking Systems: Some Comparisons	"
.20	Banking in a Demand Economy	"
.21	Current Banking Issues	"
.22	Sources of Banking Data	"
.23	President's Report, 1975	"
.24	The Banks and the West	"
.25	Governments' Place in Bank Ownership	"
.26	CBA Factbook	"
.27	The Scotia Bank Story	SB
.28	The Story of Canada's Currency	BC
.29	The Story of Credit Unions	OCUL
.30	To The Credit of The People	OCUL
.31	The Canadian Trust Companies	TCAC

2. BANK SERVICES

Accounts, Services, Alternate Money Forms, Foreign Exchange, etc.

.01	Consumer Services	RB
.02	Scotia Bank Services for your Business	SB
.03	"22" Scotia Bank Services for you and your Family	SB
.04	Guide for our Services	PBC
.05	Here's a handy checklist	SB
.06	Scotiaclub	SB
.07	Scotia Bank Courtesy Card	SB
.08	T.D. Personal Service Plan	TD
.09	Welcome to T.D.	TD
.10	Irreplaceables – Safety Deposit Box	TD
.11	Safety Deposit Box	PBC
.12	Chargex	TD
.13	Chargex Application	TD
.14	Master Charge Application	BM
.15	Personal Money Orders	TD
.16	Bank by Mail	TD
.17	Bank by Mail	PBC
.18	Bank by Mail and Night Deposit	SB
.19	Night Depository	PBC
.20	Instabank	BM
.21	Instabank Application	BM
.22	Travellers' Cheques	TD
.23	Caribbean Currency Guide	BM
.24	Currency Guide – British Isles and Continental Europe	BM
.25	Three Ways to send Money	SB
.26	Glad to Have You with Us	TD
.27	Save a Little	TD
.28	Go Together Accounts	SB
.29	Money Handling	BM
.30	The Big Plus	SB

.31	High Growth Interest Rates	SB
.32	Go Together Accounts	SB
.33	Pass Books	SB
.34	Canada Savings Bonds	SB
.35	Canadian Olympic Coins	SB
.36	Canadian Olympic Coins New Series V (Olympic Committee)	
.37	Personalize Your Cheques	SB
.38	Personalize Your Cheques Rustic Series	PBC
.39	Personalize Your Cheques In Series	PBC

3. RETIREMENT PLANS, HOME OWNERSHIP PLANS, AND LOANS

.01	The Scotiafund Retirement Plan	SB
.02	The R.R. Deposit Plan	PBC
.03	R.R.D.P. and R.H.O.D.P. – Rules and Regulations	PBC
.04	T.D. Retirement Savings Deposits	TD
.05	R.R.D.P. Plan B	PBC
.06	T.D. Home Ownership Savings Plan	TD
.07	R.H.O.D.P. and Plan B	PBC
.08	R.H.O.S.P. – The Answer Book	SB
.09	Professional Business Loans	SB
.10	Car Loans	BM
.11	Our Money is a Good Buy	SB
.12	All Purpose Loans	PBC
.13	Your Dream	BM
.14	Vacation Home Loans	BM
.15	Recreation Loans	BM
.16	Mobile Home Financing	BM
.17	Cash Flow Loans	TD
.18	Home Improvement Loan	TD
.19	Mobile Home Financing	SB
.20	Insuring Your Loan	TD

4. MONEY MANAGEMENT

.01	Investment Management	SB
.02	Credit Today and Tomorrow	CBA
.03	Money in Business	SB
.04	Personal Finances	SB
.05	Personal Money Management	TD
.06	Banking is Everybody's Business	SB
.07	Budget Book	SB
.08	Your Money Matters Kit	RB
.09	Opportunities This Way	SB
.10	The Consumer Interest	CFCS
.11	Canadian Consumer Credit Fact Book	CFCS
.12	Money Management Library	MMI
.13	Money Management Filmstrip Library	MMI

SUGGESTED REFERENCES

Dale Seymour et al, *Aftermath Series* (Setsco Educational Ltd.)
Steve and Janis Marcys, *Mathimagination* (Setsco Educational Ltd.)
Harold Jacobs, *Mathematics: A Human Endeavor* (W. H. Freeman and Co.)